Def. Vector Addition, Vector Substraction, and Scalar Multipication. A scalar is a real number. Consider vectors: $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ in $\mathbb{R}^3$. Consider a scalar $k \in \mathbb{R}$.

- $\vec{u} + \vec{v} \overset{\text{def}}{=} \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$ (Vector Addition)
- $k\vec{u} \overset{\text{def}}{=} \langle ku_1, ku_2, ku_3 \rangle$ (Scalar Multipication)
- $\vec{u} - \vec{v} \overset{\text{def}}{=} \vec{u} + ((-1)\,\vec{v}) \overset{\text{so}}{=} \langle u_1, u_2, u_3 \rangle + \langle -v_1, -v_2, -v_3 \rangle \overset{\text{so}}{=} \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$

∘. A typical format of an exam problem.
Instructions for **1** and **2**. Put your solution IN the box. Show your work BELOW/NEAR the box.
Find the following vectors. Express answer in component form, e.g., $\langle 1, 2 \rangle$ or $\langle 1, 2, 3 \rangle$.

1. Let $\vec{u} = \langle 4, 3 \rangle$ and $\vec{v} = \langle 1, -2 \rangle$.

1.1. $\frac{1}{2}\vec{u} =$ ☐

1.2. $-\vec{v} =$ ☐

1.3. $\vec{u} + \vec{v} =$ ☐

1.4. $\vec{u} - \vec{v} =$ ☐

1.5. $\vec{u}\,\vec{v} =$ Silly Prof.! We can NOT multiply vectors !!

2. Let $\vec{u} = \langle 2, 2, 1 \rangle$ and $\vec{v} = \langle -2, -1, 1 \rangle$.

2.1. $\frac{1}{2}\vec{u} =$ ☐

2.2. $-\vec{v} =$ ☐

2.3. $\vec{u} + \vec{v} =$ ☐

2.4. $\vec{u} - \vec{v} =$ ☐

Def. The vector $\vec{v}$ is a unit vector if and only if $\|\vec{v}\| = 1$.

Def. Standard unit (basis) vectors

$\mathbb{R}^3$. In $\mathbb{R}^3$, the standard unit vectors are:

$$\vec{\imath} \stackrel{\text{def}}{=} \langle 1,0,0\rangle \qquad \text{and} \qquad \vec{\jmath} \stackrel{\text{def}}{=} \langle 0,1,0\rangle \qquad \text{and} \qquad \vec{k} \stackrel{\text{def}}{=} \langle 0,0,1\rangle.$$

Any vector $\vec{v} = \langle v_1, v_2, v_3\rangle$ can be written as a linear combination of the standard unit vectors as follows:

$$\begin{aligned}\vec{v} &= \langle v_1, v_2, v_3\rangle \\ &= \langle v_1,0,0\rangle + \langle 0,v_2,0\rangle + \langle 0,0,v_3\rangle \\ &= v_1\langle 1,0,0\rangle + v_2\langle 0,1,0\rangle + v_3\langle 0,0,1\rangle \\ &= v_1\,\vec{\imath} + v_2\,\vec{\jmath} + v_3\,\vec{k}.\end{aligned}$$

The scalar v_1 is the i-component and v_2 is the j-component and v_3 is the k-component of the vector $\vec{v}$.
Note $v_1, v_2, v_3 \in \mathbb{R}$ while $\vec{\imath}$, $\vec{j}$, $\vec{k}$ are vectors.

$\mathbb{R}^2$. Similarily, in $\mathbb{R}^2$, the standard unit vectors are:

$$\vec{\imath} \stackrel{\text{def}}{=} \langle 1,0\rangle \qquad \text{and} \qquad \vec{\jmath} \stackrel{\text{def}}{=} \langle 0,1\rangle.$$

and any vector in $\mathbb{R}^2$ is a linear combination of the standard unit vectors: $\vec{v} = \langle v_1, v_2\rangle = v_1\,\vec{\imath} + v_2\,\vec{\jmath}$.

Rmk. If $\|\vec{v}\| \neq 0$, it is often helpful to write/think-of $\vec{v}$ as:

$$\vec{v} = \underbrace{\|\vec{v}\|}_{\text{the length of } \vec{v}} \quad \underbrace{\frac{\vec{v}}{\|\vec{v}\|}}_{\boxed{\text{a unit vector in the direction of } \vec{v} \text{ and is called the direction of } \vec{v}}} \tag{1}$$

Ex. Find the direction of the vector $\overrightarrow{AB}$ where the point $A = (1,2,3)$ and the point $B = (6,5,4)$.
Express your answer as a linear combination of the standard unit vectors.

Soln: The direction of $\overrightarrow{AB}$ is $\boxed{\qquad\qquad\qquad\qquad}$.

⚠. The direction of the vector $\overrightarrow{AB}$ is the unit vector in the same direction as $\overrightarrow{AB}$, which is $\frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}$ by (1).
Note $\overrightarrow{AB} = \langle 6-1\ ,\ 5-2\ ,\ 4-3\rangle = \langle 5,3,1\rangle$. So $\left\|\overrightarrow{AB}\right\| = \sqrt{5^2+3^2+1^3} = \sqrt{25+9+1} = \sqrt{35}$.

So $$\frac{\overrightarrow{AB}}{\left\|\overrightarrow{AB}\right\|} = \frac{\langle 5,3,1\rangle}{\sqrt{35}} = \frac{1}{\sqrt{35}}\langle 5,3,1\rangle = \left\langle \frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}} \right\rangle = \frac{5}{\sqrt{35}}\vec{\imath} + \frac{3}{\sqrt{35}}\vec{\jmath} + \frac{1}{\sqrt{35}}\vec{k}$$

Rmk. Properties of Vector (algebraic) Operations.
Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors. Let a and b be scalars (i.e., real numbers).

(1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
(2) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
(3) $\vec{u} + \vec{0} = \vec{u}$
(4) $\vec{u} + (-\vec{u}) = \vec{0}$
and
(5) $0\,\vec{u} = \vec{0}$
(6) $1\,\vec{u} = \vec{u}$
(7) $a\,(b\vec{u}) = (ab)\,\vec{u}$
(8) $a\,(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$
(9) $(a+b)\,\vec{u} = a\vec{u} + b\vec{u}$