Completing the Square in Circle Equations

Any equation of the form $Ax^2 + Ay^2 + Bx + Cy + D = 0$ produces a circle — as long as both squared terms have the *same* coefficient. If the coefficients differ, you get an ellipse instead.

The problem is that this general form doesn't tell you much at a glance. To find the circle's center and radius, you need to rewrite it in **standard form**:

$$(x-h)^2 + (y-k)^2 = r^2$$

Here (h, k) is the center and r is the radius. The technique for getting there is called **completing the square**.

How Completing the Square Works

Before applying it to circles, it's worth understanding what completing the square actually does.

When you expand a perfect square like $(x+6)^2$, you get:

$$(x+6)^2 = x^2 + 12x + 36$$

Notice the pattern: the constant (36) is always $\left(\frac{\text{linear coefficient}}{2}\right)^2$. Here, $\left(\frac{12}{2}\right)^2 = 6^2 = 36$. That's the whole trick — if you know the linear coefficient, you can figure out exactly what constant you'd need to make a perfect square.