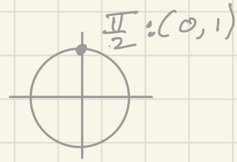
Ex 1 Thm 1 / Thm 2.

Let $W = x^2 y + y + xz$ and: $x = \cos\theta$, $y = \sin\theta$, $z = \theta^2$.

1.1 Express $\dfrac{dw}{d\theta}$ as a function of θ.

1.2 Evaluate $\left.\dfrac{dw}{d\theta}\right|_{\theta = \frac{\pi}{2}}$



Soln

1.1 Th $w = f(x, y, z)$

$$\frac{dw}{d\theta} = \frac{\partial w}{\partial x}\frac{dx}{d\theta} + \frac{\partial w}{\partial y}\frac{dy}{d\theta} + \frac{\partial w}{\partial z}\frac{dz}{d\theta}$$

$$= (2xy + z)(-\sin\theta) + (x^2 + 1)(\cos\theta) + (x)(2\theta)$$

want θ's

$$= (2\cos\theta\sin\theta + \theta^2)(-\sin\theta) + (\cos^2\theta + 1)(\cos\theta) + (\cos\theta)2\theta$$

$$= \boxed{-2\cos\theta\sin^2\theta - \theta^2\sin\theta + \cos^3\theta + \cos\theta + 2\theta\cos\theta}$$

1.2 $$\frac{dw}{d\theta}\left(\frac{\pi}{2}\right) = (-2)(0)(1)^2 - \left(\frac{\pi}{2}\right)^2(1) + 0^3 + 0 + 2\left(\frac{\pi}{2}\right)(0)$$

$$= 0 - \frac{\pi^2}{4} + 0 + 0 + 0$$

$$= \boxed{\frac{-\pi^2}{4}}$$

Ex 2 Thm 3 (and : Cor 4, Cor 5)

Let $w = x^4 y + y^2 z^3$ where

$$x = rse^t$$
$$y = rs^2 e^{-t}$$
$$z = r^2 s \sin t.$$

Evaluate $\frac{\partial w}{\partial s}$ when: $r=2$, $s=1$, $t=0$.

Soln. TL: $w = f(x, y, z)$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial s}$$

$$= (4x^3 y)(re^t) + (x^4 + 2yz^3)(2rse^{-t}) + (3y^2 z^2)(r^2 \sin t)$$

and $(r,s,t) = (2,1,0)$ get $x = 2 \cdot 1 \cdot e^0 = 2$
$y = 2\, 1^2\, e^{-0} = 2$ so
$z = 2^2\, 1 \sin 0 = 0$

$$\left.\frac{\partial w}{\partial s}\right|_{(r,s,t)=(2,1,0)} =$$

since $z=0$

$$= (2^2 \cdot 2^3 \cdot 2)(2 \cdot e^0) + (2^4 + 0)(2 \cdot 2 \cdot 1\, e^0) + 0 \text{ (who cares)}$$

$$= 2^7 + 2^6 = 2^6 (2+1) = 64(3) = \boxed{192}$$

Implicit Differentiation Examples

Ex 3 Find $\frac{dy}{dx}$ if $x^3+y^3=6xy$.

Soln. $x^3+y^3=6xy \iff x^3+y^3-6xy=0$.

So let $F(x,y)=x^3+y^3-6xy$

$$\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{3x^2+0-6y}{0+3y^2-6x}=-\frac{3x^2-6y}{3y^2-6x}=\boxed{-\frac{x^2-2y}{y^2-2x}}$$

Ex 4 Find $\frac{\partial z}{\partial x}$ if $x^3+y^3+z^3+6xyz=1$.

Soln. $x^3+y^3+z^3+6xyz=1 \iff x^3+y^3+z^3+6xyz-1=0$.

So let $F(x,y,z)=x^3+y^3+z^3+6xyz-1$

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{3x^2+0+0+6yz+0}{0+0+3z^2+6xy+0}$$

$$=\boxed{-\frac{x^2+2yz}{z^2+2xy}}$$