NAME: KEY **PIN:** 17.

This quiz is

due at the beginning of recitation on Monday October 26.

Instructions: For each problem, you are given a series. For each problem, check the appropriate box to indicate whether the given series is absolutely convergent, conditionally convergent, or divergent. Also indicate your reasoning in the space provided below the given series. Specifically specify which test(s) you are using. Show all your work (e.g., if you need to check that something is continuous and it is obviously continuous, indicate that one needs to check continuity but it is obviously continuous). A correctly checked box without appropriate explanation will receive no points. There are 6 problems on this quiz.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{7n+1}{2^n}$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

$a_n = \frac{7n+1}{2^n}$

Try the Ratio Test so $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{7(n+1)+1}{2^{n+1}} \cdot \frac{2^n}{7n+1} = \frac{7n+8}{7n+1} \cdot \frac{2^n}{2^n \cdot 2^1}$$

$$= \frac{7n+8}{7n+1} \cdot \frac{1}{2^1} \xrightarrow{n\to\infty} \frac{7}{7} \cdot \frac{1}{2^1} = \frac{1}{2} = \rho.$$

Since $0 \le \rho < 1$, the series is abs. conv. by the ratio test.

2. $\sum_{n=1}^{\infty} \frac{n!}{(-5)^n}$

- [ ] absolutely convergent
- [ ] conditionally convergent
- [x] divergent

$a_n = \frac{(-1)^n \, n!}{5^n}$

Hint: $(-5)^n = [(-1)\ (5)]^n = (-1)^n\, 5^n$.

The factorial (!) suggests ratio test.

So we consider $\rho = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$.

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)!}{5^{n+1}} \cdot \frac{5^n}{n!} = \frac{n!\,(n+1)}{n!} \cdot \frac{5^n}{5^n \cdot 5^1} = \frac{n+1}{5} \xrightarrow{n \to \infty} \infty.$$

So $\rho = \infty$.

Since $1 < \rho \leq \infty$, the ratio test gives that

$\sum a_n$ diverges.

4. $\sum_{n=1}^{\infty} \frac{(7n)^n}{(5n+3)^n}$

- [ ] absolutely convergent
- [ ] conditionally convergent
- [x] divergent

note if $n \geq 1$, then $a_n = \frac{7n}{5n+3} \geq 0$

Since

$$a_n = \frac{(7n)^n}{(5n+3)^n} = \left(\frac{7n}{5n+3}\right)^n$$

we'll try the root test which considers

$$\rho = \lim_{n \to \infty} |a_n|^{1/n}.$$

So

$$|a_n|^{1/n} = \left[\left(\frac{7n}{5n+3}\right)^n\right]^{1/n} = \frac{7n}{5n+3} \xrightarrow{n \to \infty} \boxed{\frac{7}{5} = \rho.}$$

Since $1 < \rho \leq \infty$, the root test gives that $\sum a_n$ diverges

3. $\sum_{n=1}^{\infty} \frac{(-5)^n}{n!}$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

$a_n = \frac{(-1)^n 5^n}{n!}$

The factorial (!) sugguests trying ratio test.

So we consider $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{5^{n+1}}{5^n} \frac{n!}{(n+1)!} = \frac{5^n \cdot 5}{5^n} \frac{n!}{n!(n+1)} = \frac{5}{n+1} \xrightarrow{n\to\infty} 0,$$

So $\boxed{\rho = 0}$

Since $0 \le \rho < 1$, the ratio test says that

$\sum a_n$ is abs. conv.

5. $\sum_{n=2}^{\infty} (-1)^n \frac{1}{(\ln n)^n}$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

$a_n = \frac{(-1)^n}{(\ln n)^n}$

Since $|a_n|^{1/n}$ is not to messy, let's try the ratio test where $\rho = \lim_{n\to\infty} |a_n|^{1/n}$.

So

$$|a_n|^{1/n} = \left|\frac{(-1)^n}{(\ln n)^n}\right|^{1/n} = \left(\frac{1}{\ln n}\right)^{n \cdot \frac{1}{n}} = \frac{1}{\ln n} \xrightarrow{n\to\infty} \boxed{0 = \rho}$$

Since $0 \le \rho < 1$, the ratio test tells us that $\sum a_n$ is abs. conv.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n^n}$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

$$a_n = \frac{2^{3n+1}}{n^n}.$$

Since

$$|a_n| = \frac{2^{3n+1}}{n^n} = \frac{2^{3n}\cdot 2}{n^n} = 2\left(\frac{8}{n}\right)^n,$$

let's try the ratio test where $\rho = \lim\limits_{n\to\infty} |a_n|^{1/n}$.

So

$$|a_n|^{1/n} = \left[2\cdot\left(\frac{8}{n}\right)^n\right]^{1/n} = 2^{\frac{1}{n}}\cdot\frac{8}{n} \xrightarrow{n\to\infty} 2^0\cdot 0 = 1\cdot 0 = 0.$$

So $\lim\limits_{n\to\infty} |a_n|^{1/n} = \boxed{0 = \rho}$.

Since $0 \le \rho < 1$, the ratio test gives us that Σa_n is abs. conv.