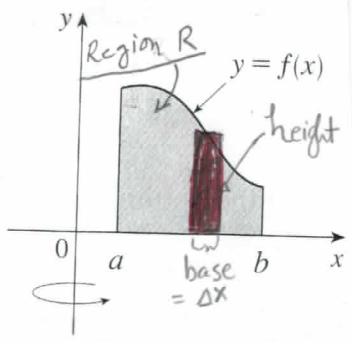
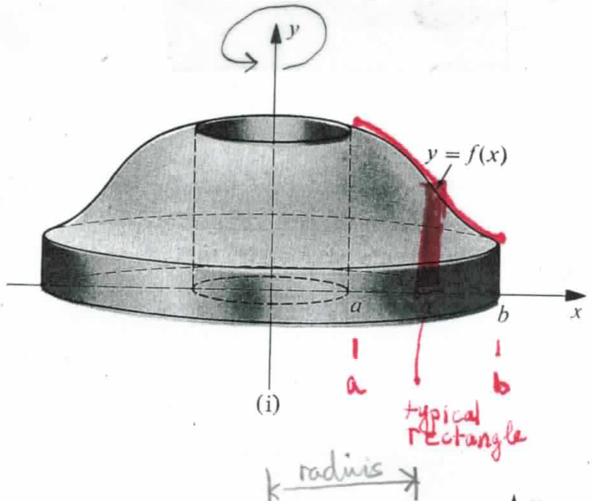
# § 6.3 Volume by (Cylindrial) Shell Method.

**Goal:** Find the volume of a $\underbrace{\boxed{\text{solid of revolution}}}$ S of R., using Shell Method.

**Visualize:** what is going on with the help of the below pictures, which were taken from: our textbook (p. 433-4), Swokowski (p 290) & our Volume Summary handout (p.3).

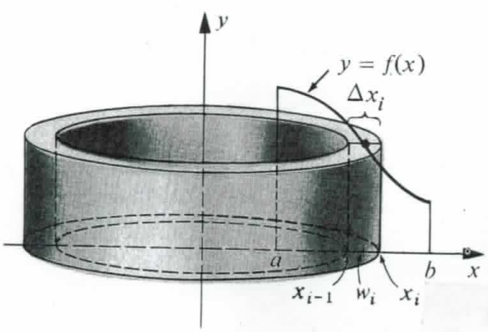
**Rotate** the region R about the x-axis & find Volume of S of R.



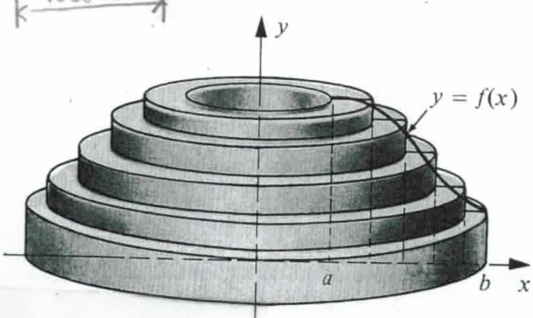




(i)

On this page, "radius" is really "average radius". On next page, we will expand on this.

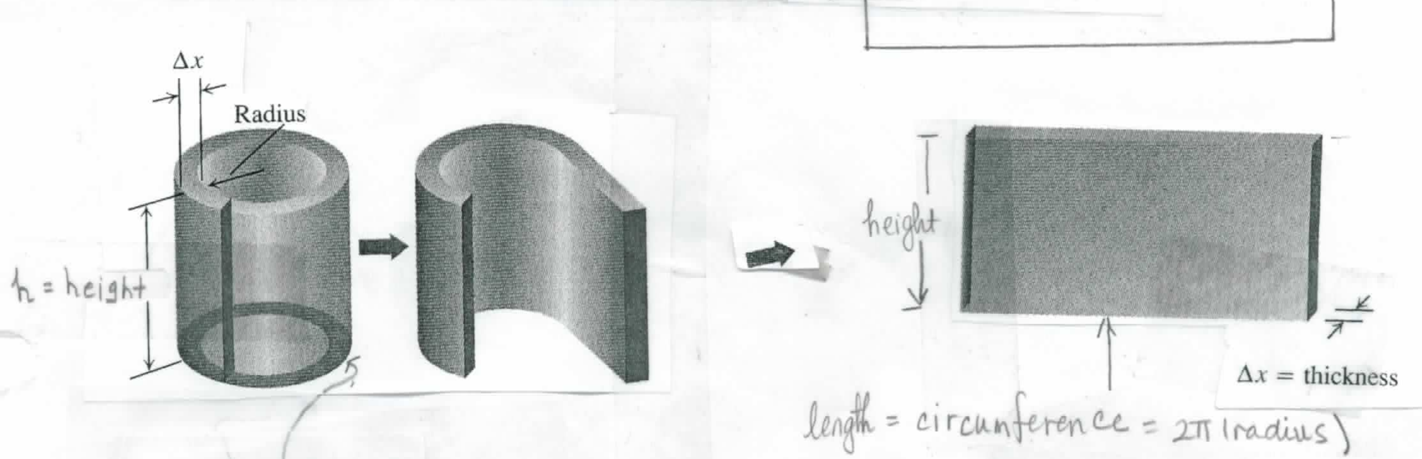



(ii)




(iii) (img: do culers here)

Imagine cutting and unrolling a cylindrical shell to get a flat (nearly) rectangular solid.




circumference = $2\pi \cdot$ radius

$\Delta x$ = thickness

length = circumference = $2\pi$ (radius)

Volume $= 2\pi(\text{radius})(\text{height})(\text{thickness})$

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Another point of view in how to find the Volume of a shell.

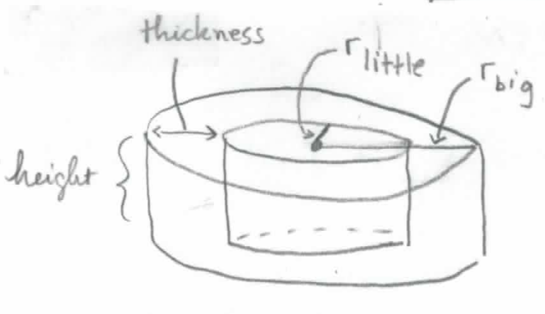
When using the Shell Method, a "typical element" is a

shell, or hair curler or what's between 2 tuna cans

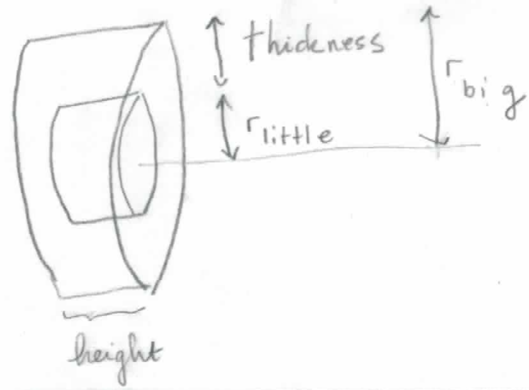
want his volume

found his volume by cutting along a side & laying him "flat"

let's find this volume.




or




$$\boxed{\text{Volume of a typical shell} = 2\pi\,(\text{average radius})\,(\text{height})\,(\text{thickness})}$$

↓ why?

$$\text{Volume} \underset{\text{washer method}}{\overset{\text{know}}{=}} \pi\left(r_{big}^2 - r_{little}^2\right)(\text{height}) \overset{(A)}{=} 2\pi \underbrace{\left(\frac{r_{big}+r_{little}}{2}\right)}_{\text{average radius}} \underbrace{\left(r_{big}-r_{little}\right)}_{\text{thickness}} (\text{height})$$

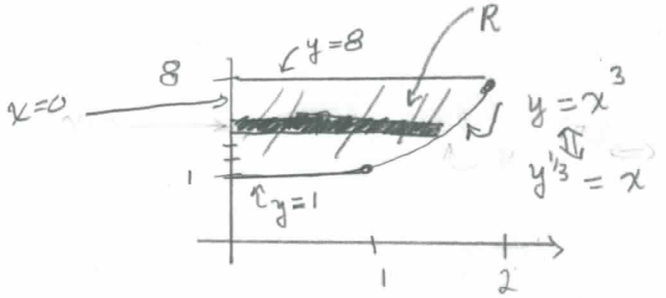
This is the only formula you need to memorize .... if you understand the theory

Theory / idea of the Shell Method – is the same as the Theory / idea of the Disk / Washer Method ... except .....

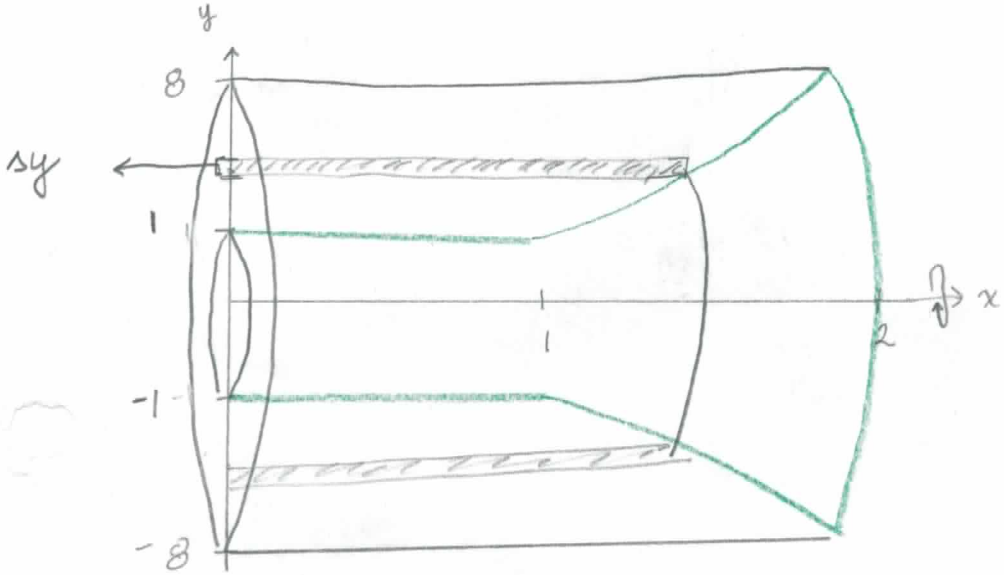
with the Shell Method we need to partition the opposite axis as we would with the Disk/Washer Method.

Ex 1  Let's do Ex 2b from § 6.2 via Shell Method.

Recall in Ex 2b we started with the region R (below) in the xy-plane




and revolved it around the x-axis to get




- Using Disk/Washer Method: partition axis of revolution = x-axis

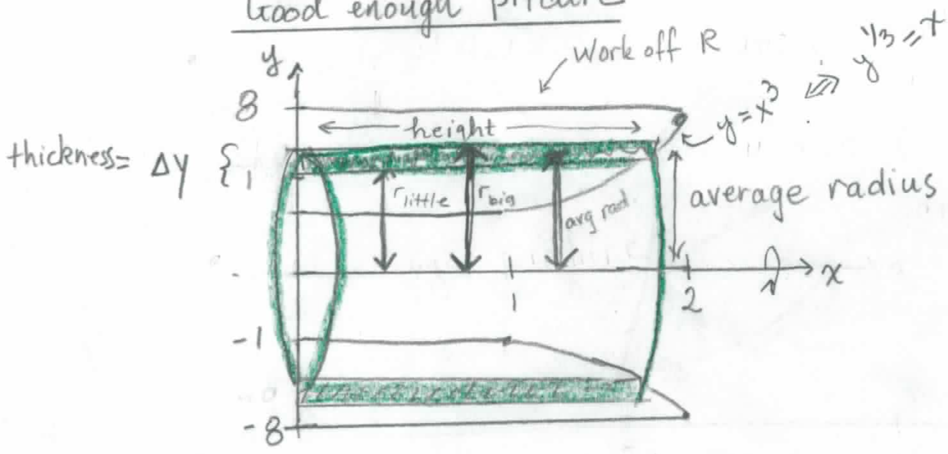
$\Downarrow$

- Using Shell Method partition y-axis

$\Downarrow$

Δy, dy, all y's

Good enough picture




Volume of typical shell

$= 2\pi(\text{avg. radius})(\text{height})(\text{thickness})$

$= 2\pi(\ y\ )(y^{1/3})(\Delta y)$

$\stackrel{A}{=} 2\pi y^{4/3} \Delta y$

$$\text{Volume} = \int_{y=1}^{y=8} 2\pi y^{4/3}\, dy \stackrel{C}{=} \frac{762\pi}{7}$$

Ex 2 The region R bounded by the graphs of ← Typical wording on our homework & exams

$y = x^2$ and $y = x + 2$

is revolved about the line $x = 3$.

Express the volume V of the resulting solid as a definite integral

$$\boxed{V = \int_{?}^{?} (?)\, d?}$$

① Points of intersection of $y = x^2$ and $y = x + 2$ :

$x^2 = x + 2 \iff x^2 - x - 2 = 0 \iff (x+1)(x-2) = 0 \iff x = -1, 2$.

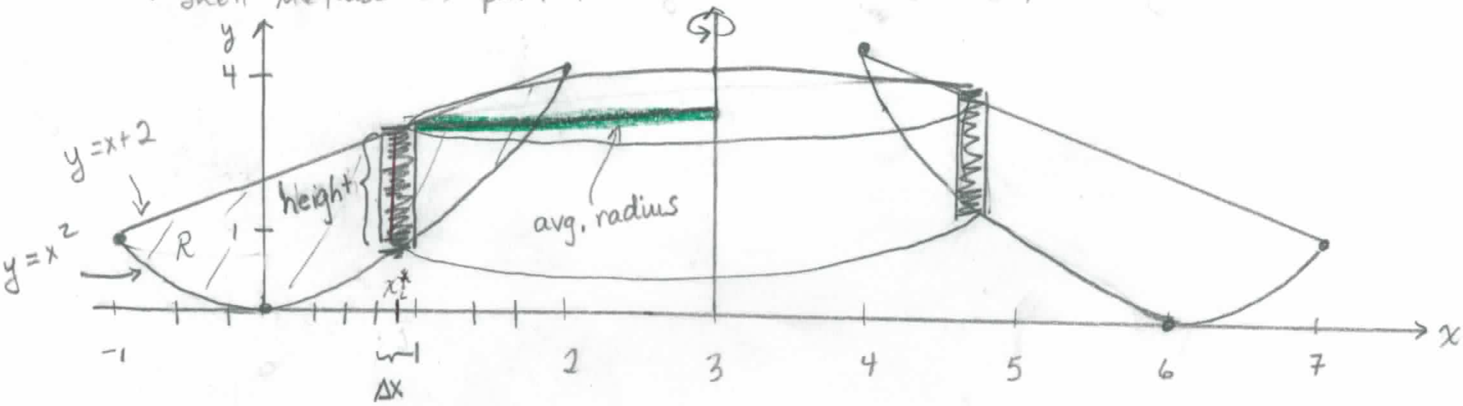
Pts of $\cap$ : $(-1, 1)$ and $(2, 4)$

② Sketch a good enough picture. What is good enough?

But first – what axis do we partition?

• Disk/Washer Method [line $x = 3$] $\parallel$ [y-axis] $\Rightarrow$ partition y-axis

• Shell Method $\Rightarrow$ partition x-axis $\Rightarrow$ $\Delta x$, $dx$, all $x$'s.




Volume of typical shell $= 2\pi\,(\text{avg. radius})(\text{height})(\text{thickness})$

$= 2\pi\,(3 - x)\,[(x+2) - (x^2)]\,(\Delta x)$

$= 2\pi\,(3 - x)\,(x + 2 - x^2)\,(\Delta x)$

$$\boxed{\text{Volume} = \int_{x=-1}^{x=2} 2\pi\,(3-x)\,(x+2-x^2)\,dx}$$

← please put in box up there

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# Shell Method - Recap

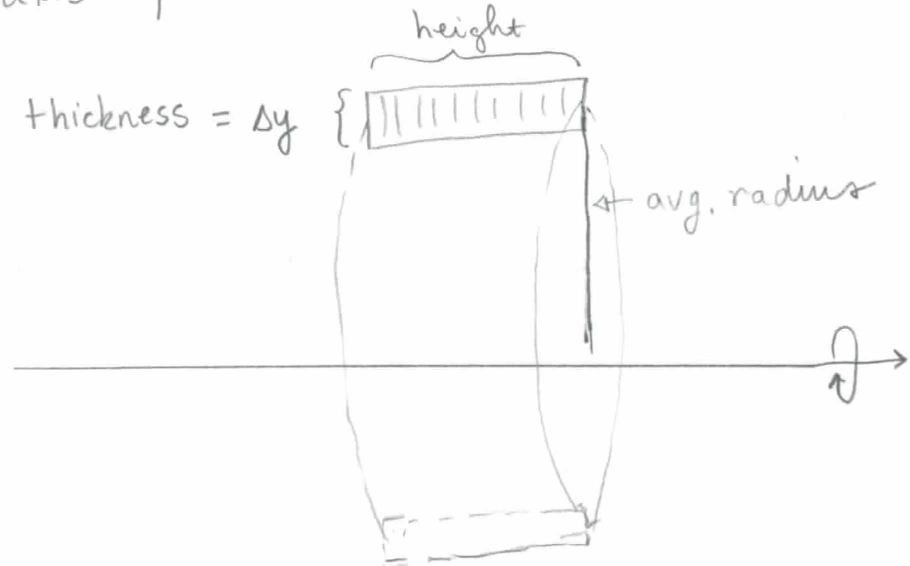
base of the "typical rectangle"

$$\text{Volume of typical shell} = (2\pi)(\text{average radius})(\text{height})\underbrace{(\text{thickness})}$$

avg radius = distance from the axis of revolution to the typical rectangle

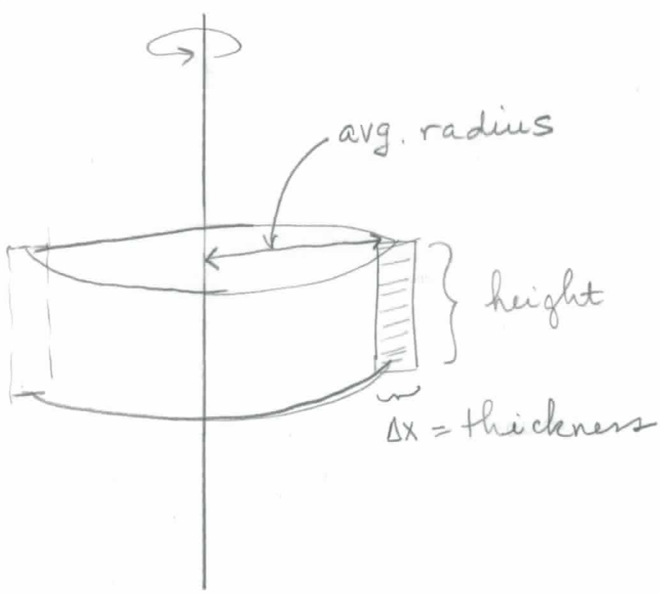
height of shell = height of "typical rectangle"

Case axis of revolution is (∥-to) x-axis. ⇒ partition y-axis





Case. axis of revolution is (∥-to) y-axis ⇒ partition x-axis

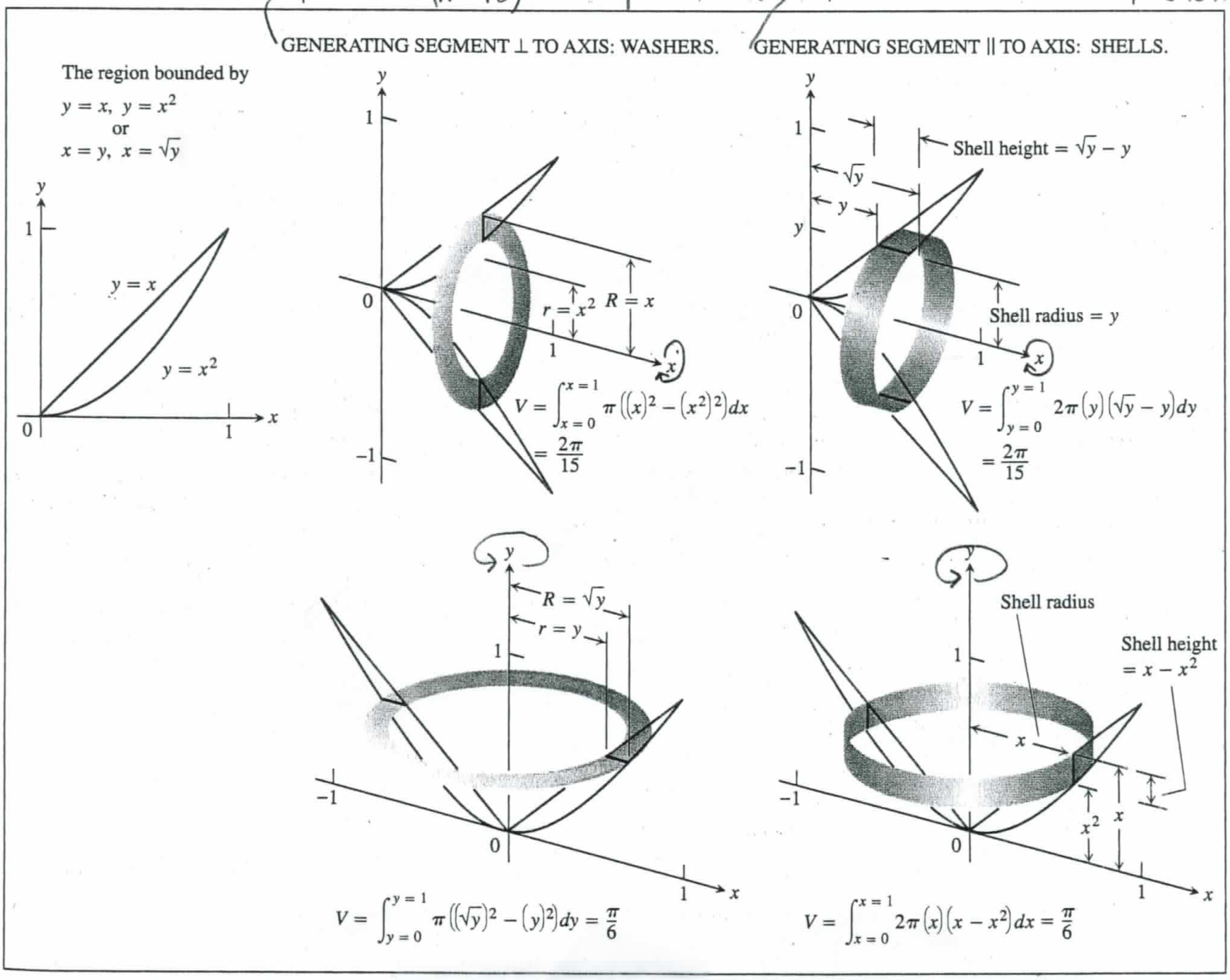



# Recap

## Disk/Washer vs. Shell (a helpful table)

**Table 5.1** Washers vs. shells

partition (|| to) axis of revolution    parition ⊥ to axis of revolution




- As a review, you should go back and reread the "Volume of Solids of Revolution" handout (all of it) and make sure it makes sense.

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