§ 6.2 Volume by Disk & Washer Method

(We will not cover Volume by slicing)

Handout : § 6.2, 6.3 Volume of Solids of Revolution

Go over page 1 & 2/3rd of page 2.

Theory/idea of the Disk/Washer Method

→ no formula needed - just understand the idea!

- Want volume
- partition the major (i.e x or y) axis that is ∥ (parallel) to axis of revolution
- Spin a typical rectangle abt. axis of revolution to get
  a typical element = [a disk (no hole) or a washer (hole)]
- Find volume of a typical element
- Use ↑ to express the (true) volume as an integral

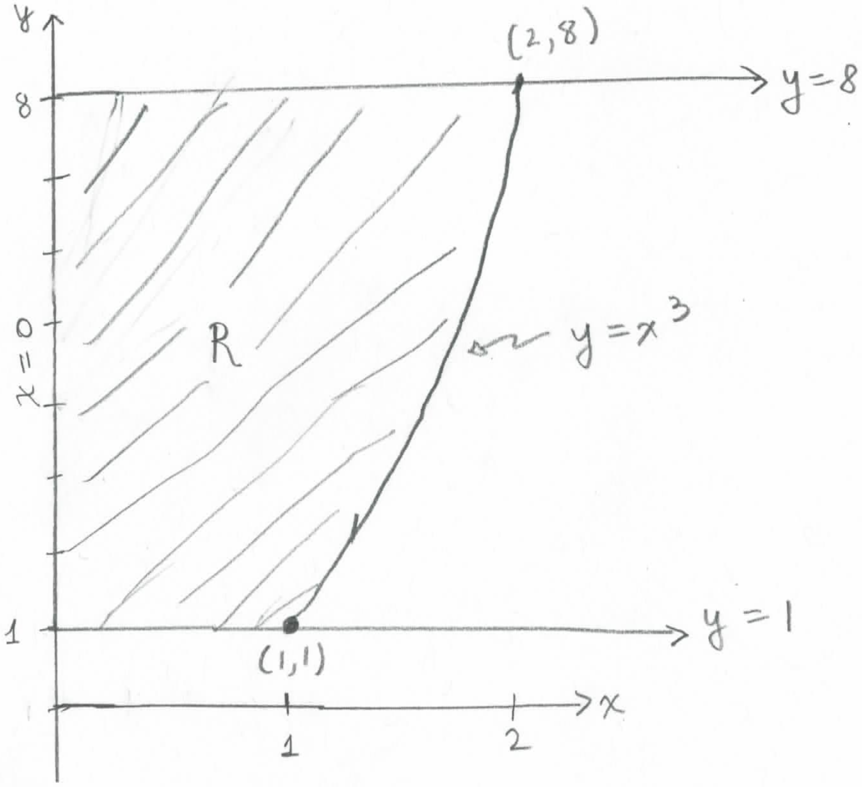
6.8

Ex 0. Let R be the region bounded by:

the y-axis, $y = x^3$, $y = 1$, $y = 8$.

$\Updownarrow$ i.e. $x = 0$ $\quad$ $\Updownarrow$ i.e. $y^{1/3} = x$

- Make a rough sketch of R.



- We will use this same region R thoughout this lecture on the disk/washer method.

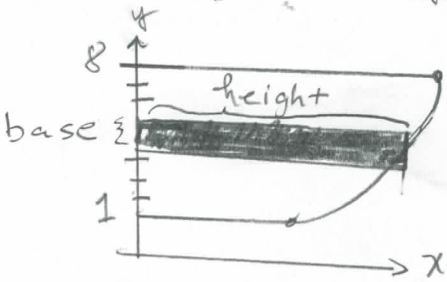
- A common shorthand:
  w.r.t. = with respect to.

Ex 1a Let A be the area of the region R (from Ex 0).

Express A as an integral $\boxed{\text{w.r.t. } y}$

$\Downarrow$

$\boxed{\text{partition } y\text{-axis} \Rightarrow \Delta y,\ dy,\ \text{all } y\text{'s}}$



Area of typical "element"

= Area of rectangle

= (height) (base)

$= \quad x \quad\quad \Delta y$ ← want only y's

$= \quad y^{1/3} \quad\quad \Delta y$

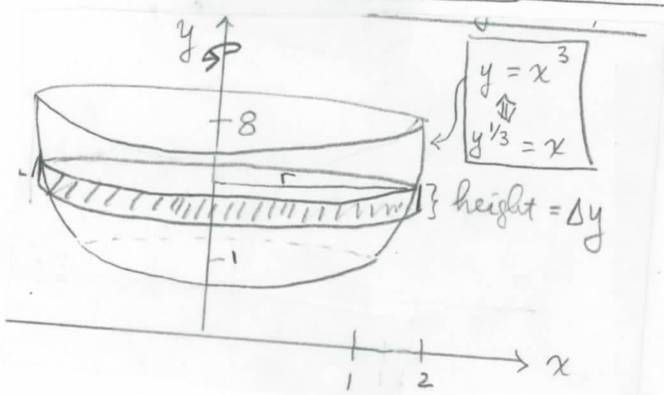
So $\boxed{A = \int_{y=1}^{y=8} y^{1/3}\, dy.}$

What is good enough?
1. Sketch in R and a "typical rectangle."
2. Sketch in the reflection of R & typ. rectangle over axis of revolution
3. Form typical element

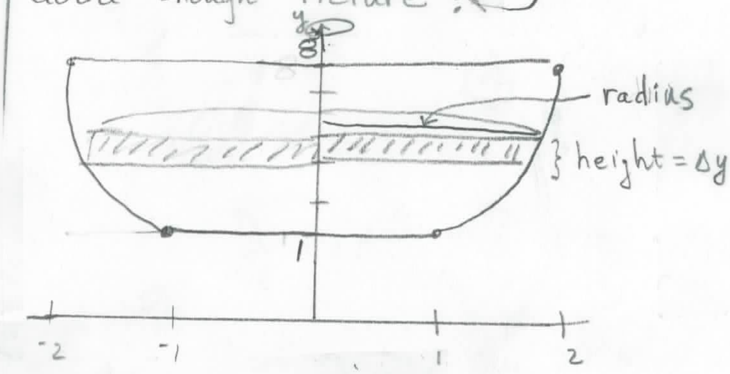
Ex 1b. Express the volume V of the solid of revolution obtained by taking the region R (from Ex 0) & revolving it about the y-axis, using the disk/washer method.

$\Downarrow$

$\boxed{\text{partition } y\text{-axis} \Rightarrow \Delta y,\ dy,\ \text{all } y\text{'s}}$



Good Enough Picture :



- no hole $\Rightarrow$ disk method $\Rightarrow$ typical element is a disk
  ↳ i.e. tuna can
- Volume of typical element = (area base) (height)

$$= \pi r^2 h = \pi \left[y^{1/3}\right]^2 (\Delta y) = \pi y^{2/3} \Delta y$$

So $\boxed{V = \int_{y=1}^{y=8} \pi y^{2/3}\, dy}$ $\quad$ B.T.W. $\frac{93\pi}{5}$.

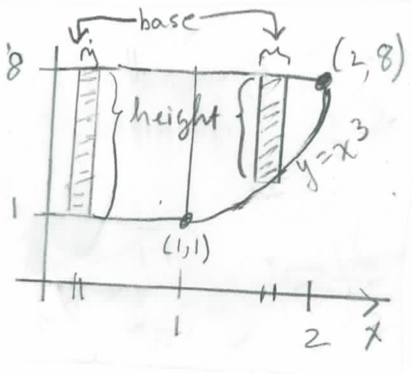
6.10

Ex 2a. Let A be the area of the region R (from Ex 0).

Express A as integral(s) $\boxed{\text{w.r.t. } x}$

$\Downarrow$

$\boxed{\text{partition } x\text{-axis} \Rightarrow \Delta x,\ dx,\ \text{all } x\text{'s.}}$



area of "typical element

$$= \text{area of typical rectangle} = (\text{height})(\text{base})$$

$$= \begin{cases} (8-1)\ \Delta x & \text{if } 0 \le x \le 1 \\ (8-x^3)\ \Delta x & \text{if } 1 \le x \le 2. \end{cases}$$

So $\boxed{A = \int_{x=0}^{x=1} 7\,dx + \int_{x=1}^{x=2} (8-x^3)\,dx}$

- Compare with Ex 1a.

Ex 2b Express the volume V of the solid of revolution obtained by taking the region R (from Ex 0) & revolving it about the x-axis, using the disk/washer method.

$\Downarrow$

$\boxed{\text{partition } x\text{-axis} \Rightarrow \Delta x,\ dx,\ \text{all } x\text{'s.}}$

- Compare: in Ex 1b, revolved R about y-axis

  in Ex 2b " x-axis.

- Look at Ex 2a just above & think about what the solid of revolution (abt x-axis) will look like.

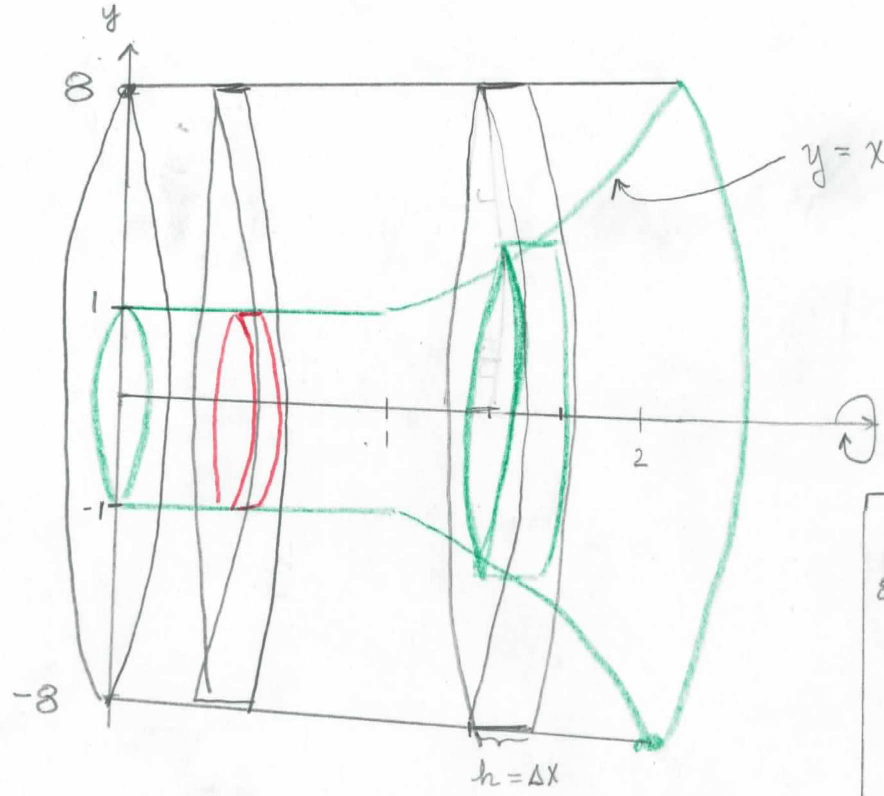
  See the hole? Let's look at the Volume of Solid of Revolution handout, last 1/3 of page 2 (about the Washer Method.

6.11

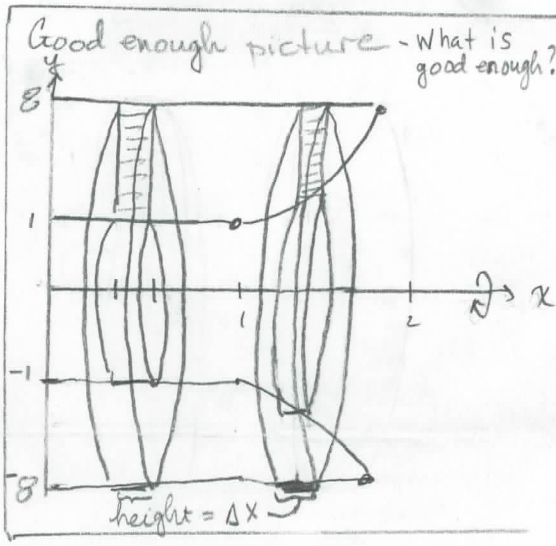
Ex 2b - continued. Revolving about the x-axis.

⇓

partition x-axis



r_{little} r_{big}



- do you see the hole ⇒ Washer Method
- partition x-axis & for typical element
  - ↳ washer = tuna can$_{big}$ − tuna can$_{little}$
- Volume of typical element = (Volume of tuna can$_{big}$) − (Volume of tuna can$_{little}$)

$$= \pi r_{big}^2 h - \pi r_{little}^2 h = \pi (r_{big}^2 - r_{little}^2) h$$

think of Ex2a

$$= \begin{cases} \pi (8^2 - 1^2)\,\Delta x & \text{if} \quad 0 \le x \le 1 \\ \pi (8^2 - (x^3)^2)\,\Delta x & \text{if} \quad 1 \le x \le 2 \end{cases}$$

- $V = \int_{x=0}^{x=1} \pi (8^2 - 1^2)\,dx + \int_{x=1}^{x=2} \pi (8^2 - (x^3)^2)\,dx$

$$= 63\pi \int_{x=0}^{x=1} dx + \pi \int_{x=1}^{x=2} (64 - x^6)\,dx \overset{W}{=} \frac{762\pi}{7}$$

important - always work off R, not the reflection of R.

Ex 3 Find the volume of the solid of revolution obtained by revolving the region R (of Ex 0) about the line $x=4$, using the disk/washer method.

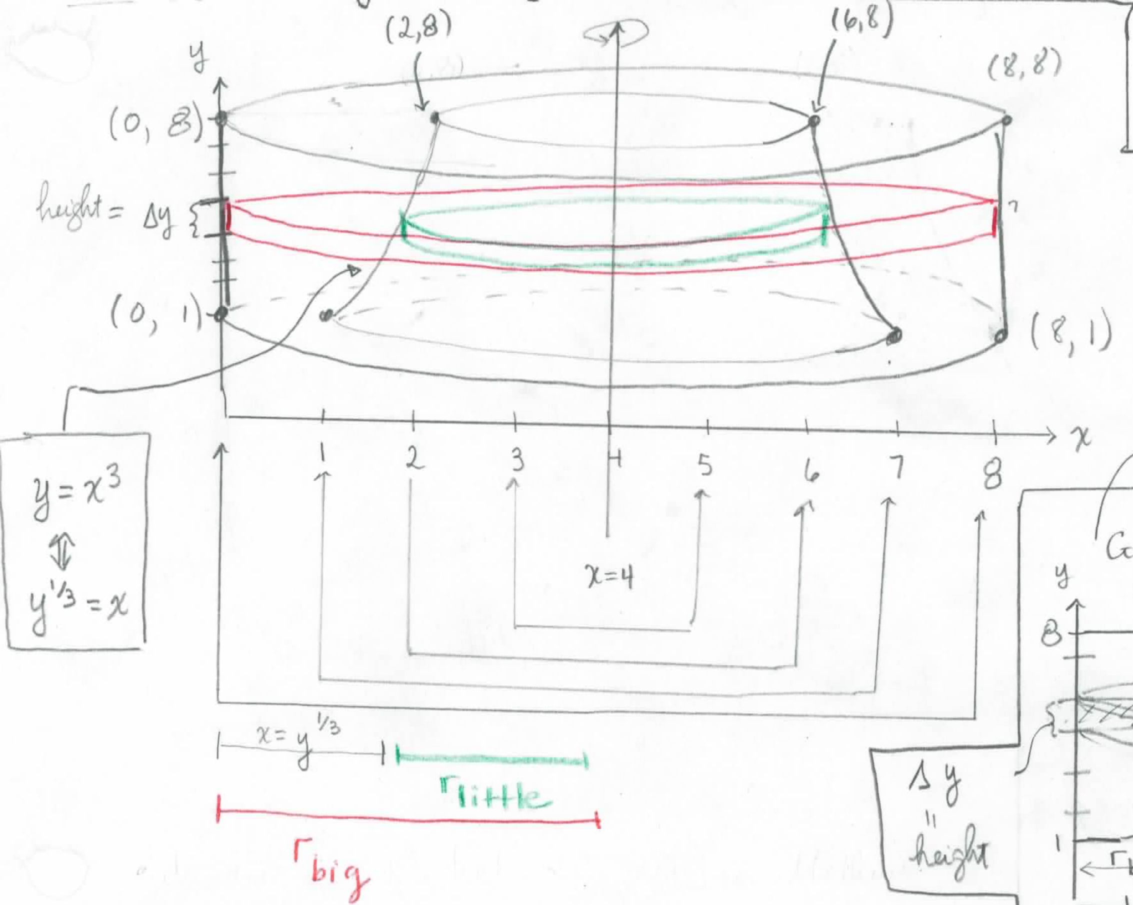
⇓

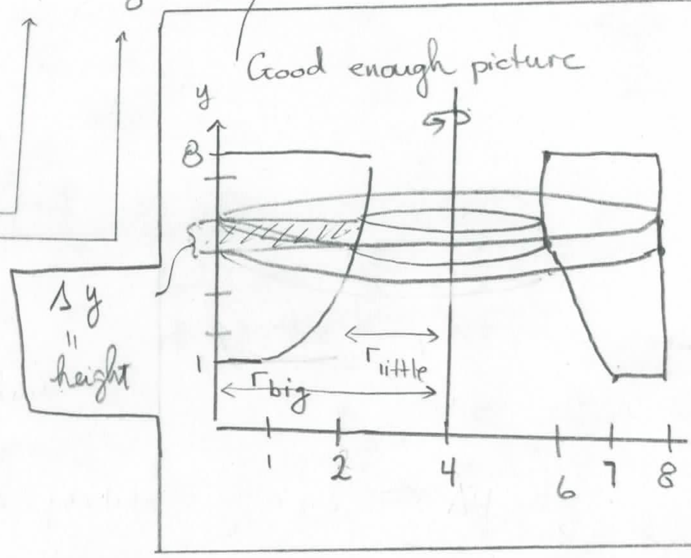
$x=4$ is $\parallel$ to y-axis so partion y-axis

⇓

Δy, dy, all y's

what is good enough?





- with hole ⇒ washer method
- [line $x=4$] is parallel to [y-axis] ⇒ partition y-axis ⇒ Δy, dy
- let's draw in a typical element = washer = tuna can$_{big}$ − tuna can$_{little}$
- Volume of typical element $= \pi\left[r_{big}^2 - r_{little}^2\right](\text{height})$

  ↓ work off "R", not reflection of R

  $= \pi\left[(4)^2 - (4-y^{1/3})^2\right](\Delta y)$

  $\overset{(A)}{=} \pi\left[8y^{1/3} - y^{2/3}\right]\Delta y$

- Volume $= \displaystyle\int_{y=1}^{y=8} \pi\left(8y^{1/3} - y^{2/3}\right)dy \overset{(c)}{=} \frac{357\pi}{5}$