A Short Review of Math 141.

① See the handout "Math 141 Review", which is posted on the course homepage. The first 3 ½ pages consists of formulas for Math 141 that you need to know for Math 142. The latter part of page 4 consists of formulas we will learn in Math 142.

② Recall Part 1 of the Fundamental Theorem of Calculus. (FTC).

Let $f: [a,b] \to \mathbb{R}$ be a continuous function. Then

$$\frac{d}{dx} \int_a^x f(t)\, dt = \boxed{} \quad \text{for } x \in (a,b)$$

What goes in the $\boxed{}$? Well, $f(x)$. Neat.

③ Exercise § 5.3 # 12. (Changed a little)

Find the derivative of the function $G(s) = \int_s^1 \cos\sqrt{t}\, dt$.

Note that G is a function of s so it is understood we want to find $\frac{d}{ds} G(s)$.

$$G'(s) = \frac{d}{ds} \int_s^1 \cos\sqrt{t}\, dt$$

$$= -\frac{d}{ds} \int_1^s \cos\sqrt{t}\, dt \qquad \text{(why?)}$$

$$= -\cos\sqrt{s}. \qquad \text{(FTC)}.$$

(4) Exercise § 5.3 # 18. (Changed a little)

Find the derivative of the function $y = \int_0^{e^x} \sin^3 t \, dt$.

- Note we could also phrase this as:
- Find the derivative of the function $F(x) = \int_0^{e^x} \sin^3 t \, dt$.

the variable is x so want the derivative wrt x.

- Want to use FTC but need a variable up here at $\int_0^{e^x}$

so $\boxed{\text{let } u = e^x}$ & apply chain rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

- So

$$\frac{d}{dx} \int_0^{e^x} \sin^3 t \, dt = \frac{d}{dx} \int_0^{u} \sin^3 t \, dt$$

$$= \left[\frac{d}{du} \int_0^{u} \sin^3 t \, dt \right] \cdot \left[\frac{d}{dx} u \right] \qquad \text{(CR)}$$

$$= \left[\sin^3 u \right] \left[\frac{d}{dx} u \right] \qquad \text{(FTC)}$$

$$= \left[\sin^3 (e^x) \right] \cdot e^x \qquad (u = e^x).$$

$$= e^x \sin^3 (e^x).$$

(5) Next Let's review Integration by Substitution, Which is from § 5.5 (from Math 141).

(6) Exercise § 5.5 # 20. Evaluate the integral. Here, a and b are constants and $a \neq 0$.

- $$\int \frac{dx}{ax+b} = \int \frac{\frac{1}{a}\,du}{u} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln|u| + C$$

$$\boxed{\begin{aligned} u &= ax+b \\ du &= a\,dx \\ \tfrac{1}{a}\,du &= dx \end{aligned}} \qquad = \boxed{\frac{1}{a}\ln|ax+b| + C}$$

- Here is another way - really the same - but faster.

$$\boxed{\begin{aligned} u &= ax+b \\ du &= \boxed{a}\,dx \end{aligned}}$$

want $\boxed{a}$ here so put it here & then $\frac{1}{a}$ outside integral sign

$$\int \frac{dx}{ax+b} = \frac{1}{a}\int \frac{\boxed{a}\,dx}{ax+b} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln|u| + C = \boxed{\frac{1}{a}\ln|ax+b| + C}$$

- What if $a = 0$? Well

$$\int \frac{dx}{ax+b} \overset{a=0}{=} \int \frac{dx}{b} = \frac{1}{b}\int dx = \frac{1}{b}x + C = \boxed{\frac{x}{b} + C}$$

(7) Exercise § 5.5 # 22. Evaluate the integral.

$$\int \sqrt{x}\,\sin(1+x^{3/2})\,dx = \int \left[\sin(1+x^{3/2})\right]\left[x^{1/2}\,dx\right]$$

$$\boxed{\begin{aligned} u &= 1+x^{3/2} \\ du &= \tfrac{3}{2}x^{1/2}\,dx \end{aligned}} \qquad = \frac{2}{3}\int \left[\sin(1+x^{3/2})\right]\left[\frac{3}{2}x^{1/2}\,dx\right]$$

see previous example

$$= \frac{2}{3}\int \sin u\,du = -\frac{2}{3}\cos u + C = \boxed{-\frac{2}{3}\cos(1+x^{3/2}) + C.}$$

continued →

⑦ continued. Next <u>check your answer</u>!

How? Just verify that

$$\text{" } D_x \text{ (answer)} = \text{integrand "}.$$

Def. The <u>integrand</u> of $\int f(x)\,dx$ is $f(x)$

$$D_x\left[-\frac{2}{3}\cos\left(1+x^{3/2}\right)+C\right]$$

$$= -\frac{2}{3} D_x \cos\left(1+x^{3/2}\right) + \cancel{D_x C}^{\to 0}$$

CR ↓

$$= \left(-\frac{2}{3}\right) -\sin\left(1+x^{3/2}\right) \cdot D_x\, x^{3/2}$$

$$= \frac{2}{3} \sin\left(1+x^{3/2}\right) \cdot \frac{3}{2} x^{1/2}$$

$$= \sqrt{x}\,\sin\left(1+x^{3/2}\right). \quad ✓$$

Home work

① Go to the course homepage at

http://www.math.sc.edu/~girardi/w142.html

② Link to "Homework"

③ Link to "Review of Calculus I"

④ Do all the problems listed under "A review of Math 141."

⑤ If you need, do the other review problems listed. In the next Chapter, you must know trigonometry and integration by substitution.