

$$\int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{2u du}{u(1+u^2)} = 2 \int \frac{du}{1+u^2} = 2 \tan^{-1} u + C$$

$$\begin{aligned} u &= x^{1/2} \\ u^2 &= x \\ 2u du &= dx \end{aligned}$$

$$= 2 \arctan \sqrt{x} + C$$

$$\int (\sin x)(\sec x) dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{du}{u}$$

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \end{aligned}$$

$$= -\ln |u| + C = -\ln |\cos x| + C$$

$$\text{or } +\ln |\cos x|^{-1} + C = \ln \frac{1}{|\cos x|} + C = \ln |\sec x| + C$$

$$\int x \tan^2 x dx = x \tan x - x^2 - \int (\tan x - x) dx$$

$$\begin{aligned} u &= x & dv &= \tan^2 x dx = (\sec^2 x - 1) dx \\ du &= dx & v &= (\tan x - x) \end{aligned}$$

$$= x \tan x - x^2 - \ln |\sec x| + \frac{x^2}{2} + C$$

$$= x \tan x - \frac{x^2}{2} + \ln |\cos x| + C$$

$$\int \ln(1+x) dx = x \ln(1+x) - \int \frac{x}{1+x} dx \stackrel{LD}{=} x \ln(1+x) - \int \frac{1+x-1}{1+x} dx$$

$$\begin{aligned} u &= \ln(1+x) & dv &= dx \\ du &= \frac{dx}{1+x} & v &= x \end{aligned}$$

$$= x \ln(1+x) - \int \left(1 - \frac{1}{1+x}\right) dx$$

$$= x \ln(1+x) - x + \ln(1+x) + C$$

$$= (x+1) \ln(1+x) - x + C$$

$$\text{or } u = \ln(1+x) \quad dv = dx$$

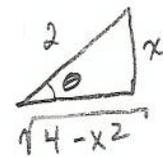
$$du = \frac{1}{1+x} dx \quad v = 1+x$$

$$\int \ln(1+x) dx = (1+x) \ln(1+x) - \int \frac{1+x}{1+x} dx$$

$$= (1+x) \ln(1+x) - \int dx$$

$$= (1+x) \ln(1+x) - x + C$$

(26) $\int \frac{x^2}{\sqrt{4-x^2}} dx$ $\rightarrow \sin \theta = \frac{x}{2}$



$x = 2 \sin \theta$
 $dx = 2 \cos \theta d\theta$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2 \theta} = 2\sqrt{1-\sin^2 \theta} = 2 \cos \theta$

$\int \frac{x^2 dx}{\sqrt{4-x^2}} = \int \frac{4 \sin^2 \theta \cdot 2 \cos \theta d\theta}{2 \cos \theta} = 4 \int \sin^2 \theta d\theta$

$= 4 \cdot \frac{1}{2} \int (1 - \cos 2\theta) d\theta = 2 \int d\theta - 2 \int \cos(2\theta) d\theta$

$= 2\theta - \sin 2\theta + C = 2\theta - 2 \sin \theta \cos \theta + C$

$= 2 \sin^{-1} \left(\frac{x}{2} \right) - 2 \left(\frac{x}{2} \right) \left(\frac{\sqrt{4-x^2}}{2} \right) + C$

$= \boxed{2 \sin^{-1} \left(\frac{x}{2} \right) - \frac{x \sqrt{4-x^2}}{2} + C}$

(35) $\int \sec^3 x \tan^3 x dx = \int \sec^2 x \tan^2 x (\sec x \tan x dx)$

$u = \tan x$
 $du = \sec^2 x dx$

get in terms of v

or

$v = \sec x$
 $dv = \sec x \tan x dx$

$= \int \sec^2 x (\sec^2 x - 1) (\sec x \tan x dx)$

$= \int v^2 (v^2 - 1) dv = \int (v^4 - v^2) dv = \frac{v^5}{5} - \frac{v^3}{3} + C$

$= \boxed{\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C}$

$$\int x^2 \tan^{-1} x \, dx = \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \int \frac{x^3}{1+x^2} \, dx \quad (36)$$

$u = \tan^{-1} x \quad dv = x^2 \, dx$ $du = \frac{dx}{1+x^2} \quad v = \frac{x^3}{3}$	$x^2 + 1 \overline{) \begin{array}{r} x \\ x^3 + 0x^2 + 0x + 0 \\ \underline{x^3} \\ -x \\ \underline{-x} \\ 0 \end{array}}$
$\frac{x^3}{1+x^2} = \frac{x(x^2+1) - x}{(x^2+1)} = x - \frac{x}{x^2+1}$	

$$= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \int \left(x - \frac{x}{x^2+1} \right) dx$$

$$= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \int x \, dx + \frac{1}{3} \int \frac{x \, dx}{x^2+1}$$

$$= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \frac{x^2}{2} + \frac{1}{6} \ln(x^2+1) + C$$

$$\int \frac{x^4 + 2x + 2}{x^4(x+1)} \, dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+1} \quad \text{Note } x^4 = (x-0)^4$$

$$\Rightarrow \frac{x^4 + 2x + 2}{x^4(x+1)} = \frac{Ax^3(x+1) + Bx^2(x+1) + Cx(x+1) + D(x+1) + Ex^4}{x^4(x+1)}$$

$$\Rightarrow x^4 + 2x + 2 = Ax^3(x+1) + Bx^2(x+1) + Cx(x+1) + D(x+1) + Ex^4$$

$$x=0 \Rightarrow 2 = D$$

$$x=-1 \Rightarrow 1 = E$$

$$x^4 : 1 = A + E \rightarrow A = 0$$

$$x^3 : 0 = A + B$$

$$x^2 : 0 = B + C \rightarrow B = 0$$

$$x : 2 = C + D \rightarrow C = 0$$

$$\text{constant} : 2 = D$$

(47) Continued

$$\int \frac{x^4 + 2x + 2}{x^5 + x^4} dx = \int \left(\frac{2}{x^4} + \frac{1}{x+1} \right) dx$$

$$= \int 2x^{-4} dx + \int \frac{dx}{x+1} = \frac{2x^{-3}}{-3} + \ln|x+1| + c$$

~~47~~ (50) $\int \frac{x}{x^4 + 4x^2 + 8} dx$ complete square $\int \frac{x}{(x^2+2)^2 + 4} dx$

$$\begin{aligned} \boxed{u = x^2 + 2} & \quad = \frac{1}{2} \int \frac{du}{u^2 + 2^2} = \frac{1}{2} \cdot \frac{1}{2} \tan^{-1} \frac{u}{2} + c \\ \boxed{du = 2x dx} & \quad = \frac{1}{4} \tan^{-1} \left(\frac{x^2 + 2}{2} \right) + c \end{aligned}$$

(or) Integrand involves $(x^2+2)^2 + 4$ i.e. $u^2 + a^2 \Rightarrow u = a \tan \theta$

$$\begin{aligned} \text{So } \boxed{x^2 + 2 = 2 \tan \theta} & \quad \rightarrow \tan \theta = \frac{x^2 + 2}{2} \\ \boxed{2x dx = 2 \sec^2 \theta d\theta} & \quad \\ \boxed{(x^2 + 2)^2 + 4 = 4 \tan^2 \theta + 4 = 4(\tan^2 \theta + 1) = 4 \sec^2 \theta} & \quad \end{aligned}$$

$$\begin{aligned} \Rightarrow \int \frac{x dx}{x^4 + 4x^2 + 8} &= \int \frac{x dx}{(x^2 + 2)^2 + 4} = \int \frac{\sec^2 \theta d\theta}{4 \sec^2 \theta} = \frac{1}{4} \int d\theta \\ &= \frac{1}{4} \theta + c = \frac{1}{4} \tan^{-1} \left(\frac{x^2 + 2}{2} \right) + c \end{aligned}$$

$$\textcircled{71} \quad \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} = \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)^2} = \frac{Ax + B}{(x^2 + 1)^1} + \frac{Cx + D}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)^2} = \frac{(Ax + B)(x^2 + 1) + (Cx + D)}{(x^2 + 1)^2}$$

$$\Rightarrow x^3 + x^2 + 2x + 1 = (Ax + B)(x^2 + 1) + (Cx + D)$$

- x^3 : 1 = A
- x^2 : 1 = B
- x : 2 = A + C \rightarrow C = 1
- constant : 1 = B + D \rightarrow D = 0

$$\int \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} dx = \int \left(\frac{x + 1}{x^2 + 1} + \frac{x}{(x^2 + 1)^2} \right) dx$$

$$= \frac{1}{2} \int \frac{2x dx}{x^2 + 1} + \int \frac{dx}{x^2 + 1} + \frac{1}{2} \int (x^2 + 1)^{-2} (2x dx)$$

$$= \frac{1}{2} \ln(x^2 + 1) + \tan^{-1}(x) + \frac{1}{2} \frac{(x^2 + 1)^{-1}}{-1} + C$$