84. Let \( x = 5 \sec u \). The integral is transformed into

\[
25 \int \sec^3 u \, du;
\]

application of Formula 28 from the endpapers and subsequent resubstitution yields the answer

\[
\frac{1}{2} \left( x(x^2 - 25)^{1/2} + 25 \ln |x + (x^2 - 25)^{1/2}| \right) + C
\]

(an extra constant in the logarithmic term has been absorbed by the constant \( C \) of integration).

85. The partial fractions decomposition of the integrand is

\[
\frac{x}{x^2 + 1} - \frac{x}{(x^2 + 1)^2},
\]

and integration of these terms presents no difficulties.

86. Because \( 6x - x^2 = 9 - (x - 3)^2 \), we use the substitution \( x = 3 + 3 \sin u \). This transforms the integral into

\[
\frac{1}{3} \int \frac{1}{1 + \sin u} \, du = \frac{1}{3} \int \frac{1 - \sin u}{\cos^2 u} \, du
\]

\[
= \frac{1}{3} \int \left( \sec^2 u - \frac{\sin u}{\cos^2 u} \right) \, du = \frac{1}{3} \left( \tan u - \frac{1}{\cos u} \right) + C
\]

\[
= \frac{-1 + \sin u}{3 \cos u} + C = \frac{-x - 6}{3(6x - x^2)^{1/2}} + C.
\]

87. Let \( x = 2 \tan \xi \); this substitution transforms the integral into

\[
\int \left( \frac{1}{2} \cos u + \frac{3}{2} \sin u \right) \, du,
\]

9.M: 527
and the rest is routine.

88. Use integration by parts with \( u = \ln x \), \( dv = x^{3/2} \) \( dx \). The antiderivative is

\[
\frac{2}{25} x^{5/2} (-2 + 5 \ln x) + C.
\]

89. If \( u = 1 + \sin^2 x \), then the integral becomes

\[
\frac{1}{2} \int \sqrt{u} \, du,
\]

and there are no further difficulties.

90. Let \( u = \sqrt{\sin x} \). The integral becomes

\[
\int 2e^u \, du = 2e^u + C = 2 \exp(\sqrt{\sin x}) + C.
\]

91. By parts: A good choice is to let \( u = x \) and \( dv = e^x \sin x \) \( dx \). It turns out that one must antidifferentiate both \( e^x \sin x \) and \( e^x \cos x \), but here Formulas 67 and 68 of the endpapers may be used, or integration by parts will suffice for each.

92. By parts: Let \( u = x^{3/2} \), \( dv = x^{1/2} \exp(x^{3/2}) \) \( dx \). The antiderivative is

\[
\frac{2}{3} (x^{3/2} - 1) \exp(x^{3/2}) + C.
\]

93. Let \( u = \arctan x \), \( dv = (x - 1)^{-3} \) \( dx \). Next, after the integration by parts, one confronts the integral

\[
\frac{1}{2} \int \frac{1}{(1 + x^2)(x - 1)^2} \, dx.
\]

The partial fractions decomposition of the integrand is

9.M: 528
\[
\frac{1}{2} \left( \frac{x}{1 + x^2} - \frac{1}{x - 1} + \frac{1}{(x - 1)^2} \right).
\]

The rest is routine.

94. Use integration by parts, with \( u = \ln(1 + \sqrt{x}) \) and \( dv = \frac{dx}{x} \). If you choose \( v = x - 1 \), certain difficulties are skirted, and the resulting antiderivative is

\[
(x - 1) \ln(1 + \sqrt{x}) - \frac{1}{2} x + \sqrt{x} + C.
\]

95. Because \( 3 + 6x - 9x^2 = 4 - (3x - 1)^2 \), we use the substitution

\[
x = \frac{1}{3} (1 + 2 \sin u).
\]

The integral is thereby transformed into

\[
\frac{1}{9} \int (11 + 4 \sin u) \, du,
\]

and the rest is standard.

96. Let \( u = \tan \frac{\xi}{2} \). The integral becomes

\[
\int \frac{2}{u^2 + 4u + 3} \, du = \int \left( \frac{1}{u + 1} - \frac{1}{u + 3} \right) \, du
\]

\[
= \ln \left| \frac{u + 1}{u + 3} \right| + C = \ln \left| \frac{1 + \tan(\xi/2)}{3 + \tan(\xi/2)} \right| + C.
\]

The transformations shown in the solution of Problem 16 of Section 9.8 allow you to write this answer in the form

\[
\ln \left| \frac{1 + \sin \xi + \cos \xi}{3 + \sin \xi + 3 \cos \xi} \right| + C.
\]

97. Multiply numerator and denominator by \( \cos \xi + 1 \). Because
\[ \cos^2 \xi - 1 = -\sin^2 \xi, \] the integral becomes

\[ \int (-\sin \xi \cos \xi - \sin \xi) \, d\xi \]

\[ = \frac{1}{2} \cos^2 \xi + \cos \xi + C. \]

98. Use integration by parts with \( u = \tan^{-1} \sqrt{x} \) and \( dv = \frac{x^{3/2}}{dx}. \) Result:

\[ \frac{2}{5} x^{5/2} \tan^{-1} \sqrt{x} - \frac{1}{5} \int \frac{x^2 - 1 + 1}{x + 1} \, dx \]

\[ = \frac{2}{5} x^{5/2} \tan^{-1} \sqrt{x} - \frac{1}{10} x^2 + \frac{1}{5} x - \frac{1}{5} \ln|x + 1| + C. \]

99. Use integration by parts with \( u = \sec^{-1} \sqrt{x}, \ dv = dx. \)

100. Let \( u = x^2. \) The integral becomes

\[ \frac{1}{2} \int \left( \frac{1 - u}{1 + u} \right)^{1/2} \, du. \]

Now let \( v^2 = \frac{1 - u}{1 + u}; \) then \( u = \frac{1 - v^2}{1 + v^2} \) and

\[ du = -\frac{4v}{(1 + v^2)^2} \, dv. \]

The integral is thereby converted into

\[ -\frac{1}{2} \int \frac{4v^2}{(1 + v^2)^2} \, dv. \]

Finally let \( v = \tan z. \) Then we obtain

9.M: 530
\[-2 \int \sin^2 z \, dz = \sin z \cos z - z + C.\]

Subsequent resubstitutions and simplifications lead to the answer:

\[\frac{1}{2} (1 - x^4)^{1/2} - \tan^{-1}\left(\frac{(1 - x^2)/(1 + x^2)}{1/2}\right) + C.\]

101. The area is

\[A = \int_0^1 2\pi \cosh^2 x \, dx\]

\[= 2\pi \left(\frac{1}{4} \sinh 2 + \frac{1}{2} - \frac{1}{4} \sinh 0 - 0\right)\]

\[= \frac{\pi}{4} (e^2 - \frac{1}{e^2} + 4) \approx 8.83865.\]

102. The length of the curve is

\[L = \int_0^1 (1 + e^{-2x})^{1/2} \, dx.\]

Now let \(e^{-x} = \tan u\). Then

\[L = \int_{x=0}^1 \frac{\sec^3 u}{\tan u} \, du\]

\[= -\int_{x=0}^1 (\csc u + \sec u \tan u) \, du\]

\[= \left[ -\ln|\csc u - \cot u| - \sec u \right]_{x=0}^1\]