Taylor Series Homework.

~~Part 2~~

Do parts (a) - (i) for the following three problems.

(1) $f(x) = \cos(17x)$	$x_0 = 0$	$J = (-\infty, \infty) = \mathbb{R}$
(2) $f(x) = (1+x)^{-3}$	$x_0 = 0$	$J = \left(0, \frac{1}{2}\right)$
(3) $f(x) = e^x$	$x_0 = 17$	$J = (16, 19)$

You might find it easier to do problems (a) - (i) in a different order. Just do what you find easiest.

Use only:

- the definition of Taylor polynominal
- the definition of Taylor series
- the theorem/error-estimate on the N^{th}-Remainder term for Taylor polynomials.

Do **NOT** use a known Taylor Series (i.e., do not use methods from section 10.10).

a. Find the following. Note the first column are functions of x and the second column are numbers.

$f^{(0)}(x) =$	$f^{(0)}(x_0) =$
$f^{(1)}(x) =$	$f^{(1)}(x_0) =$
$f^{(2)}(x) =$	$f^{(2)}(x_0) =$
$f^{(3)}(x) =$	$f^{(3)}(x_0) =$
$f^{(4)}(x) =$	$f^{(4)}(x_0) =$

b. Find the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0 in OPEN form for $N = 0, 1, 2, 3, 4$.

$P_0(x) =$
$P_1(x) =$
$P_2(x) =$
$P_3(x) =$
$P_4(x) =$

c. Find the Taylor series of $y = f(x)$ about x_0 in OPEN form.

$P_\infty(x) \;=$

d. Find the Taylor series of $y = f(x)$ about x_0 in CLOSED form.

$P_\infty(x) \;=$

e. Find the n^{th} Taylor coefficient of $y = f(x)$ about x_0.

$c_n \;=$

f. Find the interval of convergence I of the Taylor series of $y = f(x)$ about x_0. Recall, the interval of convergence is the set of points for which the series converges, either absolutely or conditionally. (Hint: use the ratio or root test and then check the endpoints.)

$I =$

g. Consider the given interval J and fix an $N \in \mathbb{N}$. Find an upper bound for the maximum of $\left|f^{(N+1)}(x)\right|$ on the interval J. You answer can have an N in it but it cannot have an: x, x_0, c. (Note that J is a subset of I but Prof. G. might have picked a smaller J than I to make the problem easier.)

$\max_{c \in J} \left|f^{(N+1)}(c)\right| \;\leq$

h. Consider the given interval J and fix an $N \in \mathbb{N}$. For each $x \in J$, find an upper bound for the maximum of $|R_N(x)|$. You answer can have an N and x in it but it cannot have an: x_0, c.

$|R_N(x)| \;\leq$

i. Carefully show that $f(x) = P_\infty(x)$ for each x in the given interval J by showing that $\lim_{N\to\infty} |R_N(x)| = 0$ for each $x \in J$.