

Taylor/Maclaurin Polynomials and Series

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0 .

Let $y = P_N(x)$ be the N^{th} -order Taylor polynomial of $y = f(x)$ about x_0 .

Let $y = R_N(x)$ be the N^{th} -order Taylor remainder of $y = f(x)$ about x_0 .

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0 .

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0 .

a. The formula for c_n is

$$c_n = \boxed{\phantom{f^{(n)}(x_0)/n!}}$$

b. In open form (i.e., with \dots and without a \sum -sign)

$$P_N(x) = \boxed{}$$

c. In closed form (i.e., with a \sum -sign and without \dots)

$$P_N(x) = \boxed{\phantom{f(x_0) + \sum_{k=1}^N \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k}}$$

d. In open form (i.e., with \dots and without a \sum -sign)

$$P_\infty(x) = \boxed{}$$

e. In closed form (i.e., with a \sum -sign and without \dots)

$$P_\infty(x) = \boxed{\phantom{f(x_0) + \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k}}$$

f. We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$$R_N(x) = \boxed{\phantom{\frac{f^{(N+1)}(c)}{(N+1)!} (x-x_0)^{N+1}}} \text{ for some } c \text{ between } \boxed{} \text{ and } \boxed{}.$$

g. A Maclaurin series is a Taylor series with the center specifically specified as $x_0 = \boxed{}$.