
Part 1 — Fill in the box

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0 .

Let $y = P_N(x)$ be the N^{th} -order Taylor polynomial of $y = f(x)$ about x_0 .

Let $y = R_N(x)$ be the N^{th} -order Taylor remainder of $y = f(x)$ about x_0 .

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0 .

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0 .

A. In open form (i.e., with \dots and without a \sum -sign)

$$P_N(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \dots + \frac{f^{(N)}(x_0)}{N!}(x - x_0)^N$$

B. In closed form (i.e., with a \sum -sign and without \dots)

$$P_N(x) = \sum_{n=0}^N \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

C. In open form (i.e., with \dots and without a \sum -sign)

$$P_\infty(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \dots$$

D. In closed form (i.e., with a \sum -sign and without \dots)

$$P_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

E. We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$$R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!} (x - x_0)^{(N+1)}$$

for some c between x and x_0 .

F. The formula for c_n is

$$c_n = \frac{f^{(n)}(x_0)}{n!}$$