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65. $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx, \quad a \neq 0$

66. $\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$

67. $\int x^m (\ln x)^n dx = \frac{x^{m+1}}{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx, \quad m \neq -1$

68. Use Example 5 to show that

$$\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx$$

$$= \begin{cases} \left(\frac{\pi}{2}\right) \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n}, & n \text{ even} \\ \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdots n}, & n \text{ odd} \end{cases}$$

69. Show that

$$\int_a^b \left(\int_x^b f(t) dt \right) dx = \int_a^b (x-a)f(x) dx.$$

70. Use integration by parts to obtain the formula

$$\int \sqrt{1-x^2} dx = \frac{1}{2} x \sqrt{1-x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx.$$

Integrating Inverses of Functions

Integration by parts leads to a rule for integrating inverses that usually gives good results:

$$\begin{aligned} \int f^{-1}(x) dx &= \int y f'(y) dy & y = f^{-1}(x), \quad x = f(y) \\ &= yf(y) - \int f(y) dy & \text{Integration by parts with} \\ &= xf^{-1}(x) - \int f(y) dy & u = y, \quad dv = f'(y) dy \end{aligned}$$

The idea is to take the most complicated part of the integral, in this case $f^{-1}(x)$, and simplify it first. For the integral of $\ln x$, we get

$$\begin{aligned} \int \ln x dx &= \int ye^y dy & y = \ln x, \quad x = e^y \\ &= ye^y - e^y + C & dy = e^y dx \\ &= x \ln x - x + C. & \end{aligned}$$

For the integral of $\cos^{-1} x$ we get

$$\begin{aligned} \int \cos^{-1} x dx &= x \cos^{-1} x - \int \cos y dy & y = \cos^{-1} x \\ &= x \cos^{-1} x - \sin y + C \\ &= x \cos^{-1} x - \sin(\cos^{-1} x) + C. \end{aligned}$$

Use the formula

$$\int f^{-1}(x) dx = xf^{-1}(x) - \int f(y) dy \quad y = f^{-1}(x) \quad (4)$$

to evaluate the integrals in Exercises 71–74. Express your answers in terms of x .

71. $\int \sin^{-1} x dx$ 72. $\int \tan^{-1} x dx$
 73. $\int \sec^{-1} x dx$ 74. $\int \log_2 x dx$

Another way to integrate $f^{-1}(x)$ (when f^{-1} is integrable, of course) is to use integration by parts with $u = f^{-1}(x)$ and $dv = dx$ to rewrite the integral of f^{-1} as

$$\int f^{-1}(x) dx = xf^{-1}(x) - \int x \left(\frac{d}{dx} f^{-1}(x) \right) dx. \quad (5)$$

Exercises 75 and 76 compare the results of using Equations (4) and (5).

75. Equations (4) and (5) give different formulas for the integral of $\cos^{-1} x$:

- a. $\int \cos^{-1} x dx = x \cos^{-1} x - \sin(\cos^{-1} x) + C$ Eq. (4)
 b. $\int \cos^{-1} x dx = x \cos^{-1} x - \sqrt{1-x^2} + C$ Eq. (5)

Can both integrations be correct? Explain.

76. Equations (4) and (5) lead to different formulas for the integral of $\tan^{-1} x$:

- a. $\int \tan^{-1} x dx = x \tan^{-1} x - \ln \sec(\tan^{-1} x) + C$ Eq. (4)
 b. $\int \tan^{-1} x dx = x \tan^{-1} x - \ln \sqrt{1+x^2} + C$ Eq. (5)

Can both integrations be correct? Explain.

Evaluate the integrals in Exercises 77 and 78 with (a) Eq. (4) and (b) Eq. (5). In each case, check your work by differentiating your answer with respect to x .

77. $\int \sinh^{-1} x dx$ 78. $\int \tanh^{-1} x dx$

8.3 Trigonometric Integrals

Trigonometric integrals involve algebraic combinations of the six basic trigonometric functions. In principle, we can always express such integrals in terms of sines and cosines, but it is often simpler to work with other functions, as in the integral

$$\int \sec^2 x dx = \tan x + C.$$

The general idea is to use identities to transform the integrals we have to find into integrals that are easier to work with.

Products of Powers of Sines and Cosines

We begin with integrals of the form

$$\int \sin^m x \cos^n x \, dx,$$

where m and n are nonnegative integers (positive or zero). We can divide the appropriate substitution into three cases according to m and n being odd or even.

Case 1 If m is odd, we write m as $2k + 1$ and use the identity $\sin^2 x = 1 - \cos^2 x$ to obtain

$$\sin^m x = \sin^{2k+1} x = (\sin^2 x)^k \sin x = (1 - \cos^2 x)^k \sin x. \quad (1)$$

Then we combine the single $\sin x$ with dx in the integral and set $\sin x \, dx$ equal to $-d(\cos x)$.

Case 2 If m is even and n is odd in $\int \sin^m x \cos^n x \, dx$, we write n as $2k + 1$ and use the identity $\cos^2 x = 1 - \sin^2 x$ to obtain

$$\cos^n x = \cos^{2k+1} x = (\cos^2 x)^k \cos x = (1 - \sin^2 x)^k \cos x.$$

We then combine the single $\cos x$ with dx and set $\cos x \, dx$ equal to $d(\sin x)$.

Case 3 If both m and n are even in $\int \sin^m x \cos^n x \, dx$, we substitute

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2} \quad (2)$$

to reduce the integrand to one in lower powers of $\cos 2x$.

Here are some examples illustrating each case.

EXAMPLE 1 Evaluate

$$\int \sin^3 x \cos^2 x \, dx.$$

Solution This is an example of Case 1.

$$\begin{aligned} \int \sin^3 x \cos^2 x \, dx &= \int \sin^2 x \cos^2 x \sin x \, dx && m \text{ is odd.} \\ &= \int (1 - \cos^2 x)(\cos^2 x)(-\sin x) \, dx && \sin x \, dx = -d(\cos x) \\ &= \int (1 - u^2)(u^2)(-du) && u = \cos x \\ &= \int (u^4 - u^2) \, du && \text{Multiply terms.} \\ &= \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C \end{aligned}$$

EXAMPLE 2 Evaluate

$$\int \cos^5 x \, dx.$$

Solution This is an example of Case 2, where $m = 0$ is even and $n = 5$ is odd.

$$\begin{aligned} \int \cos^5 x \, dx &= \int \cos^4 x \cos x \, dx = \int (1 - \sin^2 x)^2 d(\sin x) & \cos x \, dx = d(\sin x) \\ &= \int (1 - u^2)^2 \, du & u = \sin x \\ &= \int (1 - 2u^2 + u^4) \, du & \text{Square } 1 - u^2. \\ &= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C \end{aligned}$$

EXAMPLE 3 Evaluate

$$\int \sin^2 x \cos^4 x \, dx.$$

Solution This is an example of Case 3.

$$\begin{aligned} \int \sin^2 x \cos^4 x \, dx &= \int \left(\frac{1 - \cos 2x}{2}\right) \left(\frac{1 + \cos 2x}{2}\right)^2 \, dx & m \text{ and } n \text{ both even} \\ &= \frac{1}{8} \int (1 - \cos 2x)(1 + 2\cos 2x + \cos^2 2x) \, dx \\ &= \frac{1}{8} \int (1 + \cos 2x - \cos^2 2x - \cos^3 2x) \, dx \\ &= \frac{1}{8} \left[x + \frac{1}{2} \sin 2x - \int (\cos^2 2x + \cos^3 2x) \, dx \right] \end{aligned}$$

For the term involving $\cos^2 2x$, we use

$$\begin{aligned} \int \cos^2 2x \, dx &= \frac{1}{2} \int (1 + \cos 4x) \, dx \\ &= \frac{1}{2} \left(x + \frac{1}{4} \sin 4x \right). & \text{Omitting the constant of integration until the final result} \end{aligned}$$

For the $\cos^3 2x$ term, we have

$$\begin{aligned} \int \cos^3 2x \, dx &= \int (1 - \sin^2 2x) \cos 2x \, dx & u = \sin 2x, \\ &= \frac{1}{2} \int (1 - u^2) \, du = \frac{1}{2} \left(\sin 2x - \frac{1}{3} \sin^3 2x \right). & du = 2 \cos 2x \, dx \quad \text{Again omitting } C \end{aligned}$$

Combining everything and simplifying, we get

$$\int \sin^2 x \cos^4 x \, dx = \frac{1}{16} \left(x - \frac{1}{4} \sin 4x + \frac{1}{3} \sin^3 2x \right) + C.$$

Eliminating Square Roots

In the next example, we use the identity $\cos^2 \theta = (1 + \cos 2\theta)/2$ to eliminate a square root.

EXAMPLE 4 Evaluate

$$\int_0^{\pi/4} \sqrt{1 + \cos 4x} dx.$$

Solution To eliminate the square root, we use the identity

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad \text{or} \quad 1 + \cos 2\theta = 2 \cos^2 \theta.$$

With $\theta = 2x$, this becomes

$$1 + \cos 4x = 2 \cos^2 2x.$$

Therefore,

$$\begin{aligned} \int_0^{\pi/4} \sqrt{1 + \cos 4x} dx &= \int_0^{\pi/4} \sqrt{2 \cos^2 2x} dx = \int_0^{\pi/4} \sqrt{2} \sqrt{\cos^2 2x} dx \\ &= \sqrt{2} \int_0^{\pi/4} |\cos 2x| dx = \sqrt{2} \int_0^{\pi/4} \cos 2x dx \\ &= \sqrt{2} \left[\frac{\sin 2x}{2} \right]_0^{\pi/4} = \frac{\sqrt{2}}{2} [1 - 0] = \frac{\sqrt{2}}{2}. \end{aligned}$$

$\cos 2x \geq 0$ on
 $[0, \pi/4]$

Integrals of Powers of $\tan x$ and $\sec x$

We know how to integrate the tangent and secant and their squares. To integrate higher powers, we use the identities $\tan^2 x = \sec^2 x - 1$ and $\sec^2 x = \tan^2 x + 1$, and integrate by parts when necessary to reduce the higher powers to lower powers.

EXAMPLE 5 Evaluate

$$\int \tan^4 x dx.$$

Solution

$$\begin{aligned} \int \tan^4 x dx &= \int \tan^2 x \cdot \tan^2 x dx = \int \tan^2 x \cdot (\sec^2 x - 1) dx \\ &= \int \tan^2 x \sec^2 x dx - \int \tan^2 x dx \\ &= \int \tan^2 x \sec^2 x dx - \int (\sec^2 x - 1) dx \\ &= \int \tan^2 x \sec^2 x dx - \int \sec^2 x dx + \int dx \end{aligned}$$

In the first integral, we let

$$u = \tan x, \quad du = \sec^2 x dx$$

and have

$$\int u^2 du = \frac{1}{3} u^3 + C_1.$$

The remaining integrals are standard forms, so

$$\int \tan^4 x \, dx = \frac{1}{3} \tan^3 x - \tan x + x + C.$$

EXAMPLE 6 Evaluate

$$\int \sec^3 x \, dx.$$

Solution We integrate by parts using

$$u = \sec x, \quad dv = \sec^2 x \, dx, \quad v = \tan x, \quad du = \sec x \tan x \, dx.$$

Then

$$\begin{aligned} \int \sec^3 x \, dx &= \sec x \tan x - \int (\tan x)(\sec x \tan x \, dx) \\ &= \sec x \tan x - \int (\sec^2 x - 1) \sec x \, dx \quad \tan^2 x = \sec^2 x - 1 \\ &= \sec x \tan x + \int \sec x \, dx - \int \sec^3 x \, dx. \end{aligned}$$

Combining the two secant-cubed integrals gives

$$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x \, dx$$

and

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C.$$

EXAMPLE 7 Evaluate

$$\int \tan^4 x \sec^4 x \, dx.$$

Solution

$$\begin{aligned} \int (\tan^4 x)(\sec^4 x) \, dx &= \int (\tan^4 x)(1 + \tan^2 x)(\sec^2 x) \, dx \quad \sec^2 x = 1 + \tan^2 x \\ &= \int (\tan^4 x + \tan^6 x)(\sec^2 x) \, dx \\ &= \int (\tan^4 x)(\sec^2 x) \, dx + \int (\tan^6 x)(\sec^2 x) \, dx \\ &= \int u^4 \, du + \int u^6 \, du = \frac{u^5}{5} + \frac{u^7}{7} + C \quad u = \tan x, \\ &= \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C \quad du = \sec^2 x \, dx \end{aligned}$$

Products of Sines and Cosines

The integrals

$$\int \sin mx \sin nx \, dx, \quad \int \sin mx \cos nx \, dx, \quad \text{and} \quad \int \cos mx \cos nx \, dx$$

arise in many applications involving periodic functions. We can evaluate these integrals through integration by parts, but two such integrations are required in each case. It is simpler to use the identities

$$\sin mx \sin nx = \frac{1}{2} [\cos(m-n)x - \cos(m+n)x], \tag{3}$$

$$\sin mx \cos nx = \frac{1}{2} [\sin(m-n)x + \sin(m+n)x], \tag{4}$$

$$\cos mx \cos nx = \frac{1}{2} [\cos(m-n)x + \cos(m+n)x]. \tag{5}$$

These identities come from the angle sum formulas for the sine and cosine functions (Section 1.3). They give functions whose antiderivatives are easily found.

EXAMPLE 8 Evaluate

$$\int \sin 3x \cos 5x \, dx.$$

Solution From Equation (4) with $m = 3$ and $n = 5$, we get

$$\begin{aligned} \int \sin 3x \cos 5x \, dx &= \frac{1}{2} \int [\sin(-2x) + \sin 8x] \, dx \\ &= \frac{1}{2} \int (\sin 8x - \sin 2x) \, dx \\ &= -\frac{\cos 8x}{16} + \frac{\cos 2x}{4} + C. \end{aligned}$$

Exercises 8.3

Powers of Sines and Cosines

Evaluate the integrals in Exercises 1–22.

1. $\int \cos 2x \, dx$
2. $\int_0^\pi 3 \sin \frac{x}{3} \, dx$
3. $\int \cos^3 x \sin x \, dx$
4. $\int \sin^4 2x \cos 2x \, dx$
5. $\int \sin^3 x \, dx$
6. $\int \cos^3 4x \, dx$
7. $\int \sin^5 x \, dx$
8. $\int_0^\pi \sin^5 \frac{x}{2} \, dx$
9. $\int \cos^3 x \, dx$
10. $\int_0^{\pi/6} 3 \cos^5 3x \, dx$
11. $\int \sin^3 x \cos^3 x \, dx$
12. $\int \cos^3 2x \sin^5 2x \, dx$
13. $\int \cos^2 x \, dx$
14. $\int_0^{\pi/2} \sin^2 x \, dx$
15. $\int_0^{\pi/2} \sin^7 y \, dy$
16. $\int 7 \cos^7 t \, dt$
17. $\int_0^\pi 8 \sin^4 x \, dx$
18. $\int 8 \cos^4 2\pi x \, dx$
19. $\int 16 \sin^2 x \cos^2 x \, dx$
20. $\int_0^\pi 8 \sin^4 y \cos^2 y \, dy$
21. $\int 8 \cos^3 2\theta \sin 2\theta \, d\theta$
22. $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta \, d\theta$

Integrating Square Roots

Evaluate the integrals in Exercises 23–32.

23. $\int_0^{2\pi} \sqrt{\frac{1 - \cos x}{2}} \, dx$
24. $\int_0^\pi \sqrt{1 - \cos 2x} \, dx$
25. $\int_0^\pi \sqrt{1 - \sin^2 t} \, dt$
26. $\int_0^\pi \sqrt{1 - \cos^2 \theta} \, d\theta$

27. $\int_{\pi/3}^{\pi/2} \frac{\sin^2 x}{\sqrt{1 - \cos x}} \, dx$
28. $\int_0^{\pi/6} \sqrt{1 + \sin x} \, dx$

$$\left(\text{Hint: Multiply by } \sqrt{\frac{1 - \sin x}{1 - \sin x}}\right)$$
29. $\int_{5\pi/6}^\pi \frac{\cos^4 x}{\sqrt{1 - \sin x}} \, dx$
30. $\int_{\pi/2}^{3\pi/4} \sqrt{1 - \sin 2x} \, dx$
31. $\int_0^{\pi/2} \theta \sqrt{1 - \cos 2\theta} \, d\theta$
32. $\int_{-\pi}^\pi (1 - \cos^2 t)^{3/2} \, dt$

Powers of Tangents and Secants

Evaluate the integrals in Exercises 33–50.

33. $\int \sec^2 x \tan x \, dx$
34. $\int \sec x \tan^2 x \, dx$
35. $\int \sec^3 x \tan x \, dx$
36. $\int \sec^3 x \tan^3 x \, dx$
37. $\int \sec^2 x \tan^2 x \, dx$
38. $\int \sec^4 x \tan^2 x \, dx$
39. $\int_{-\pi/3}^0 2 \sec^3 x \, dx$
40. $\int e^x \sec^3 e^x \, dx$
41. $\int \sec^4 \theta \, d\theta$
42. $\int 3 \sec^4 3x \, dx$
43. $\int_{\pi/4}^{\pi/2} \csc^4 \theta \, d\theta$
44. $\int \sec^6 x \, dx$
45. $\int 4 \tan^3 x \, dx$
46. $\int_{-\pi/4}^{\pi/4} 6 \tan^4 x \, dx$
47. $\int \tan^5 x \, dx$
48. $\int \cot^6 2x \, dx$
49. $\int_{\pi/6}^{\pi/3} \cot^3 x \, dx$
50. $\int 8 \cot^4 t \, dt$

Products of Sines and Cosines

Evaluate the integrals in Exercises 51–56.

51. $\int \sin 3x \cos 2x \, dx$

52. $\int \sin 2x \cos 3x \, dx$

53. $\int_{-\pi}^{\pi} \sin 3x \sin 3x \, dx$

54. $\int_0^{\pi/2} \sin x \cos x \, dx$

55. $\int \cos 3x \cos 4x \, dx$

56. $\int_{-\pi/2}^{\pi/2} \cos x \cos 7x \, dx$

Exercises 57–62 require the use of various trigonometric identities before you evaluate the integrals.

57. $\int \sin^2 \theta \cos 3\theta \, d\theta$

58. $\int \cos^2 2\theta \sin \theta \, d\theta$

59. $\int \cos^3 \theta \sin 2\theta \, d\theta$

60. $\int \sin^3 \theta \cos 2\theta \, d\theta$

61. $\int \sin \theta \cos \theta \cos 3\theta \, d\theta$

62. $\int \sin \theta \sin 2\theta \sin 3\theta \, d\theta$

Assorted Integrations

Use any method to evaluate the integrals in Exercises 63–68.

63. $\int \frac{\sec^3 x}{\tan x} \, dx$

64. $\int \frac{\sin^3 x}{\cos^4 x} \, dx$

65. $\int \frac{\tan^2 x}{\csc x} \, dx$

67. $\int x \sin^2 x \, dx$

66. $\int \frac{\cot x}{\cos^2 x} \, dx$

68. $\int x \cos^3 x \, dx$

Applications69. **Arc length** Find the length of the curve

$y = \ln(\sec x), \quad 0 \leq x \leq \pi/4.$

70. **Center of gravity** Find the center of gravity of the region bounded by the x -axis, the curve $y = \sec x$, and the lines $x = -\pi/4, x = \pi/4$.71. **Volume** Find the volume generated by revolving one arch of the curve $y = \sin x$ about the x -axis.72. **Area** Find the area between the x -axis and the curve $y = \sqrt{1 + \cos 4x}, 0 \leq x \leq \pi$.73. **Centroid** Find the centroid of the region bounded by the graphs of $y = x + \cos x$ and $y = 0$ for $0 \leq x \leq 2\pi$.74. **Volume** Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sin x + \sec x, y = 0, x = 0$, and $x = \pi/3$ about the x -axis.

8.4 Trigonometric Substitutions

Trigonometric substitutions occur when we replace the variable of integration by a trigonometric function. The most common substitutions are $x = a \tan \theta$, $x = a \sin \theta$, and $x = a \sec \theta$. These substitutions are effective in transforming integrals involving $\sqrt{a^2 + x^2}$, $\sqrt{a^2 - x^2}$, and $\sqrt{x^2 - a^2}$ into integrals we can evaluate directly since they come from the reference right triangles in Figure 8.2.

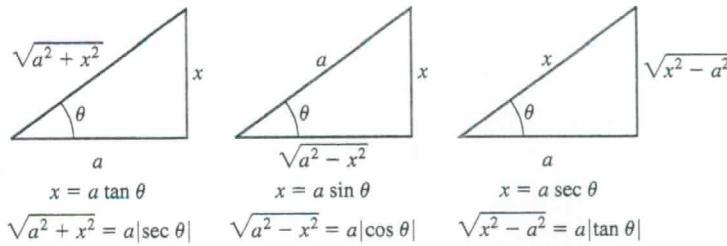


FIGURE 8.2 Reference triangles for the three basic substitutions identifying the sides labeled x and a for each substitution.

With $x = a \tan \theta$,

$$a^2 + x^2 = a^2 + a^2 \tan^2 \theta = a^2(1 + \tan^2 \theta) = a^2 \sec^2 \theta.$$

With $x = a \sin \theta$,

$$a^2 - x^2 = a^2 - a^2 \sin^2 \theta = a^2(1 - \sin^2 \theta) = a^2 \cos^2 \theta.$$

Section 8.2, pp. 467–469

1. $-2x \cos(x/2) + 4 \sin(x/2) + C$

3. $t^2 \sin t + 2t \cos t - 2 \sin t + C$

5. $\ln 4 - \frac{3}{4}$ 7. $x e^x - e^x + C$

9. $-(x^2 + 2x + 2)e^{-x} + C$

11. $y \tan^{-1}(y) - \ln \sqrt{1+y^2} + C$

13. $x \tan x + \ln |\cos x| + C$

15. $(x^3 - 3x^2 + 6x - 6)e^x + C$ 17. $(x^2 - 7x + 7)e^x + C$

19. $(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)e^x + C$

21. $\frac{1}{2}(-e^\theta \cos \theta + e^\theta \sin \theta) + C$

23. $\frac{e^{2x}}{13}(3 \sin 3x + 2 \cos 3x) + C$

25. $\frac{2}{3}(\sqrt{3s+9}e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}) + C$

27. $\frac{\pi\sqrt{3}}{3} - \ln(2) - \frac{\pi^2}{18}$

29. $\frac{1}{2}[-x \cos(\ln x) + x \sin(\ln x)] + C$

31. $\frac{1}{2} \ln |\sec x^2 + \tan x^2| + C$

33. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$

35. $-\frac{1}{x} \ln x - \frac{1}{x} + C$ 37. $\frac{1}{4}e^{x^4} + C$

39. $\frac{1}{3}x^2(x^2 + 1)^{3/2} - \frac{2}{15}(x^2 + 1)^{5/2} + C$

41. $-\frac{2}{5} \sin 3x \sin 2x - \frac{3}{5} \cos 3x \cos 2x + C$

43. $\frac{2}{9}x^{3/2}(3 \ln x - 2) + C$

45. $2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$

47. $\frac{\pi^2 - 4}{8}$ 49. $\frac{5\pi - 3\sqrt{3}}{9}$

51. $\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{x}{2} + C$

53. (a) π (b) 3π (c) 5π (d) $(2n+1)\pi$

55. $2\pi(1 - \ln 2)$ 57. (a) $\pi(\pi - 2)$ (b) 2π

59. (a) 1 (b) $(e-2)\pi$ (c) $\frac{\pi}{2}(e^2 + 9)$

(d) $\bar{x} = \frac{1}{4}(e^2 + 1)$, $\bar{y} = \frac{1}{2}(e-2)$

61. $\frac{1}{2\pi}(1 - e^{-2\pi})$ 63. $u = x^n$, $dv = \cos x \, dx$

65. $u = x^n$, $dv = e^{ux} \, dx$ 71. $x \sin^{-1} x + \cos(\sin^{-1} x) + C$

73. $x \sec^{-1} x - \ln |x + \sqrt{x^2 - 1}| + C$ 75. Yes

77. (a) $x \sinh^{-1} x - \cosh(\sinh^{-1} x) + C$
(b) $x \sinh^{-1} x - (1 + x^2)^{1/2} + C$

Section 8.3, pp. 474–475

1. $\frac{1}{2} \sin 2x + C$ 3. $-\frac{1}{4} \cos^4 x + C$

5. $\frac{1}{3} \cos^3 x - \cos x + C$

7. $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$

9. $\sin x - \frac{1}{3} \sin^3 x + C$ 11. $\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C$

13. $\frac{1}{2}x + \frac{1}{4} \sin 2x + C$ 15. $16/35$ 17. 3π

19. $-4 \sin x \cos^3 x + 2 \cos x \sin x + 2x + C$

21. $-\cos^4 2\theta + C$ 23. 4 25. 2

27. $\sqrt{\frac{3}{2}} - \frac{2}{3}$ 29. $\frac{4}{5}\left(\frac{3}{2}\right)^{5/2} - \frac{18}{35} - \frac{2}{7}\left(\frac{3}{2}\right)^{7/2}$ 31. $\sqrt{2}$

33. $\frac{1}{2} \tan^2 x + C$ 35. $\frac{1}{3} \sec^3 x + C$ 37. $\frac{1}{3} \tan^3 x + C$

39. $2\sqrt{3} + \ln(2 + \sqrt{3})$ 41. $\frac{2}{3} \tan \theta + \frac{1}{3} \sec^2 \theta \tan \theta + C$

43. $4/3$ 45. $2 \tan^2 x - 2 \ln(1 + \tan^2 x) + C$

47. $\frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x + \ln |\sec x| + C$ 49. $\frac{4}{3} - \ln \sqrt{3}$

51. $-\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$ 53. π

55. $\frac{1}{2} \sin x + \frac{1}{14} \sin 7x + C$

57. $\frac{1}{6} \sin 3\theta - \frac{1}{4} \sin \theta - \frac{1}{20} \sin 5\theta + C$

59. $-\frac{2}{5} \cos^5 \theta + C$ 61. $\frac{1}{4} \cos \theta - \frac{1}{20} \cos 5\theta + C$

63. $\sec x - \ln |\csc x + \cot x| + C$ 65. $\cos x + \sec x + C$

67. $\frac{1}{4}x^2 - \frac{1}{4}x \sin 2x - \frac{1}{8} \cos 2x + C$ 69. $\ln(1 + \sqrt{2})$

71. $\pi^2/2$ 73. $\bar{x} = \frac{4\pi}{3}$, $\bar{y} = \frac{8\pi^2 + 3}{12\pi}$

Section 8.4, pp. 479–480

1. $\ln|\sqrt{9+x^2} + x| + C$ 3. $\pi/4$ 5. $\pi/6$

7. $\frac{25}{2} \sin^{-1}\left(\frac{t}{5}\right) + \frac{t\sqrt{25-t^2}}{2} + C$

9. $\frac{1}{2} \ln \left| \frac{2x}{7} + \frac{\sqrt{4x^2-49}}{7} \right| + C$

11. $7 \left[\frac{\sqrt{y^2-49}}{7} - \sec^{-1}\left(\frac{y}{7}\right) \right] + C$ 13. $\frac{\sqrt{x^2-1}}{x} + C$

15. $-\sqrt{9-x^2} + C$ 17. $\frac{1}{3}(x^2 + 4)^{3/2} - 4\sqrt{x^2+4} + C$

19. $\frac{-2\sqrt{4-w^2}}{w} + C$ 21. $\sin^{-1} x - \sqrt{1-x^2} + C$

23. $4\sqrt{3} - \frac{4\pi}{3}$ 25. $-\frac{x}{\sqrt{x^2-1}} + C$

27. $-\frac{1}{5}\left(\frac{\sqrt{1-x^2}}{x}\right)^5 + C$ 29. $2 \tan^{-1} 2x + \frac{4x}{(4x^2+1)} + C$

31. $\frac{1}{2}x^2 + \frac{1}{2} \ln|x^2 - 1| + C$ 33. $\frac{1}{3}\left(\frac{v}{\sqrt{1-v^2}}\right)^3 + C$

35. $\ln 9 - \ln(1 + \sqrt{10})$ 37. $\pi/6$ 39. $\sec^{-1}|x| + C$

41. $\sqrt{x^2-1} + C$ 43. $\frac{1}{2} \ln |\sqrt{1+x^4} + x^2| + C$

45. $4 \sin^{-1} \frac{\sqrt{x}}{2} + \sqrt{x} \sqrt{4-x} + C$

47. $\frac{1}{4} \sin^{-1} \sqrt{x} - \frac{1}{4} \sqrt{x} \sqrt{1-x}(1-2x) + C$

49. $y = 2 \left[\frac{\sqrt{x^2-4}}{2} - \sec^{-1}\left(\frac{x}{2}\right) \right]$

51. $y = \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) - \frac{3\pi}{8}$ 53. $3\pi/4$