

Review for basic Integration Techniques : $u - du$ Sub., Parts, Trig Integral, Trig Substitution, Partial Fractions. Time to test your pattern recognition skills, which is a valuable skill no matter what your academic pursuit is. For each integration technique, look over the list of integral problems from that section and see what they have in common as to learn when the technique from that section should be applied. Then evaluate the integrals below.

Answers and Solutions to All: Answers and solutions to all can be found on the course homepage. To find: Course Homepage → Homework for Chapter on Integration → Review of Sections over Integration Techniques → look under the row for 81 Integrals.

Clarification (for hard to read/see print ones) and hints (to harder ones) are given.

1. $\int \cos x (1 + \sin^2 x) dx$

2. $\int \frac{\sin^3 x}{\cos x} dx$

3. $\int \frac{\sin x + \sec x}{\tan x} dx$

4. $\int \tan^3 \theta d\theta$

5. $\int_0^2 \frac{2t}{(t-3)^2} dt$

6. $\int \frac{x}{\sqrt{3-x^4}} dx$

7. $\int_{-1}^1 \frac{e^{\arctan y}}{1+y^2} dy$

8. $\int x \csc x \cot x dx$

9. $\int_1^3 r^4 \ln r dr$

10. $\int_0^4 \frac{x-1}{x^2-4x-5} dx$

11. $\int \frac{x-1}{x^2-4x+5} dx$

12. $\int \frac{x}{x^4+x^2+1} dx$

13. $\int \sin^3 \theta \cos^5 \theta d\theta$

14. $\int \frac{x^3}{\sqrt{1+x^2}} dx$

15. $\int \frac{dx}{(1-x^2)^{3/2}}$

16. $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}} dx$

17. $\int x \sin^2 x dx$

18. $\int \frac{e^{2t}}{1+e^{4t}} dt$

19. $\int e^{x+e^x} dx$

20. $\int e^2 dx$

21. $\int \arctan \sqrt{x} dx$

22. $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}} dx$

23. $\int_0^1 (1+\sqrt{x})^8 dx$

24. $\int \ln(x^2-1) dx$

16 reads: $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}} dx.$

Hint for 23: $u = 1 + x^{1/2}.$

25. $\int \frac{3x^2 - 2}{x^2 - 2x - 8} dx$

26. $\int \frac{3x^2 - 2}{x^3 - 2x - 8} dx$

27. $\int \frac{dx}{1 + e^x}$

28. $\int \sin \sqrt{at} dt$

29. $\int_0^5 \frac{3w - 1}{w + 2} dw$

30. $\int_{-2}^2 |x^2 - 4x| dx$

31. $\int \sqrt{\frac{1+x}{1-x}} dx$

32. $\int \frac{\sqrt{2x-1}}{2x+3} dx$

33. $\int \sqrt{3-2x-x^2} dx$

34. $\int_{\pi/4}^{\pi/2} \frac{1+4\cot x}{4-\cot x} dx$

35. $\int_{-1}^1 x^8 \sin x dx$

36. $\int \sin 4x \cos 3x dx$

37. $\int_0^{\pi/4} \cos^2 \theta \tan^2 \theta d\theta$

38. $\int_0^{\pi/4} \tan^5 \theta \sec^3 \theta d\theta$

39. $\int \frac{\sec \theta \tan \theta}{\sec^2 \theta - \sec \theta} d\theta$

40. $\int \frac{1}{\sqrt{4y^2 - 4y - 3}} dy$

41. $\int \theta \tan^2 \theta d\theta$

42. $\int \frac{\tan^{-1} x}{x^2} dx$

43. $\int e^x \sqrt{1+e^x} dx$

44. $\int \sqrt{1+e^x} dx$

45. $\int x^5 e^{-x^3} dx$

46. $\int \frac{1 + \sin x}{1 - \sin x} dx$

47. $\int x^3(x-1)^{-4} dx$

48. $\int \frac{x}{x^4 - a^4} dx$

35 reads: $\int_{-1}^1 x^8 \sin x dx$. 44 reads: $\int \sqrt{1+e^x} dx$. 45 reads: $\int x^5 e^{-x^3} dx$.

Hint for 31: conjugate - multiply the numerator and denominator by $(1+x)^{1/2}$.

49. $\int \frac{1}{x\sqrt{4x+1}} dx$

50. $\int \frac{1}{x^2\sqrt{4x+1}} dx$

51. $\int \frac{1}{x\sqrt{4x^2+1}} dx$

52. $\int \frac{dx}{x(x^4+1)}$

53. $\int x^2 \sinh mx dx$

54. $\int (x + \sin x)^2 dx$

55. $\int \frac{dx}{x + x\sqrt{x}}$

56. $\int \frac{dx}{\sqrt{x} + x\sqrt{x}}$

57. $\int x\sqrt[3]{x+c} dx$

58. $\int \frac{x \ln x}{\sqrt{x^2-1}} dx$

59. $\int \cos x \cos^3(\sin x) dx$

60. $\int \frac{dx}{x^2\sqrt{4x^2-1}}$

61. $\int \sqrt{x} e^{\sqrt{x}} dx$

62. $\int \frac{1}{x + \sqrt[3]{x}} dx$

63. $\int \frac{\sin 2x}{1 + \cos^4 x} dx$

64. $\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\sin x \cos x} dx$

65. $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

66. $\int_2^3 \frac{u^3 + 1}{u^3 - u^2} du$

Hint for 66: PFD. Do you have (strictly) bigger bottoms? The PFD of the integrand is $1 + \frac{2}{u-1} - \frac{1}{u} - \frac{1}{u^2}$.

67. $\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2} dx$ 68. $\int \frac{1}{1+2e^x - e^{-x}} dx$
69. $\int \frac{e^{2x}}{1+e^x} dx$ 70. $\int \frac{\ln(x+1)}{x^2} dx$
71. $\int \frac{x + \arcsin x}{\sqrt{1-x^2}} dx$ 72. $\int \frac{4^x + 10^x}{2^x} dx$
73. $\int \frac{1}{(x-2)(x^2+4)} dx$ 74. $\int \frac{dx}{\sqrt{x}(2+\sqrt{x})^4}$
75. $\int \frac{xe^x}{\sqrt{1+e^x}} dx$ 76. $\int (x^2 - bx) \sin 2x dx$
77. $\int \frac{\sqrt{x}}{1+x^3} dx$ 78. $\int \frac{\sec x \cos 2x}{\sin x + \sec x} dx$
79. $\int x \sin^2 x \cos x dx$ 80. $\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

81. The functions $y = e^{x^2}$ and $y = x^2 e^{x^2}$ don't have elementary antiderivatives, but $y = (2x^2 + 1)e^{x^2}$ does. Evaluate $\int (2x^2 + 1)e^{x^2} dx$.

Answers to Odd

EXERCISES 7.5 • PAGE 488

1. $\sin x + \frac{1}{3} \sin^3 x + C$
3. $\sin x + \ln |\csc x - \cot x| + C$
5. $4 - \ln 9$ 7. $e^{\pi/4} - e^{-\pi/4}$
9. $\frac{243}{5} \ln 3 - \frac{242}{25}$ 11. $\frac{1}{2} \ln(x^2 - 4x + 5) + \tan^{-1}(x - 2) + C$
13. $\frac{1}{8} \cos^8 \theta - \frac{1}{6} \cos^6 \theta + C$ (or $\frac{1}{4} \sin^4 \theta - \frac{1}{3} \sin^6 \theta + \frac{1}{8} \sin^8 \theta + C$)
15. $x/\sqrt{1-x^2} + C$
17. $\frac{1}{4} x^2 - \frac{1}{2} x \sin x \cos x + \frac{1}{4} \sin^2 x + C$
(or $\frac{1}{4} x^2 - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + C$)
19. $e^{e^x} + C$ 21. $(x+1) \arctan \sqrt{x} - \sqrt{x} + C$
23. $\frac{4097}{45}$ 25. $3x + \frac{23}{3} \ln |x-4| - \frac{5}{3} \ln |x+2| + C$
27. $x - \ln(1+e^x) + C$ 29. $15 + 7 \ln \frac{2}{7}$
31. $\sin^{-1} x - \sqrt{1-x^2} + C$
33. $2 \sin^{-1} \left(\frac{x+1}{2} \right) + \frac{x+1}{2} \sqrt{3-2x-x^2} + C$
35. 0 37. $\pi/8 - \frac{1}{4}$ 39. $\ln |\sec \theta - 1| - \ln |\sec \theta| + C$
41. $\theta \tan \theta - \frac{1}{2} \theta^2 - \ln |\sec \theta| + C$ 43. $\frac{2}{3} (1+e^x)^{3/2} + C$
45. $-\frac{1}{3} (x^3 + 1) e^{-x^3} + C$
47. $\ln |x-1| - 3(x-1)^{-1} - \frac{3}{2} (x-1)^{-2} - \frac{1}{3} (x-1)^{-3} + C$
49. $\ln \left| \frac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1} \right| + C$ 51. $-\ln \left| \frac{\sqrt{4x^2+1}+1}{2x} \right| + C$
53. $\frac{1}{m} x^2 \cosh(mx) - \frac{2}{m^2} x \sinh(mx) + \frac{2}{m^3} \cosh(mx) + C$
55. $2 \ln \sqrt{x} - 2 \ln(1+\sqrt{x}) + C$
57. $\frac{3}{7} (x+c)^{7/3} - \frac{3}{4} c(x+c)^{4/3} + C$
59. $\sin(\sin x) - \frac{1}{3} \sin^3(\sin x) + C$ 61. $2(x-2\sqrt{x}+2)e^{\sqrt{x}} + C$
63. $-\tan^{-1}(\cos^2 x) + C$ 65. $\frac{2}{3} [(x+1)^{3/2} - x^{3/2}] + C$
67. $\sqrt{2} - 2/\sqrt{3} + \ln(2+\sqrt{3}) - \ln(1+\sqrt{2})$
69. $e^x - \ln(1+e^x) + C$
71. $-\sqrt{1-x^2} + \frac{1}{2} (\arcsin x)^2 + C$
73. $\frac{1}{8} \ln |x-2| - \frac{1}{16} \ln(x^2+4) - \frac{1}{8} \tan^{-1}(x/2) + C$
75. $2(x-2)\sqrt{1+e^x} + 2 \ln \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1} + C$
77. $\frac{2}{3} \tan^{-1}(x^{3/2}) + C$
79. $\frac{1}{3} x \sin^3 x + \frac{1}{3} \cos x - \frac{1}{9} \cos^3 x + C$ 81. $xe^{x^2} + C$

Problems from *Calculus, Early Transcendental* by Stewart (6th ed.) Section 7.5 pages 488-489.