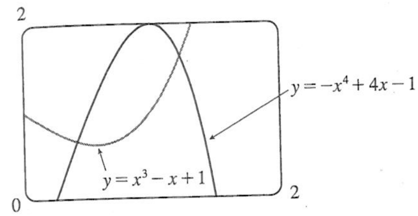
34.

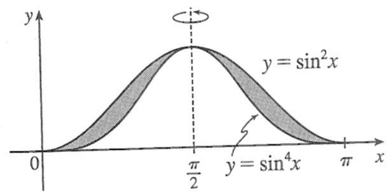


From the graph, the curves intersect at $x = a \approx 0.42$ and $x = b \approx 1.23$, with $-x^4 + 4x - 1 > x^3 - x + 1$ on the interval (a, b). So the volume of the solid obtained by rotating the region about the y-axis is

$$V = 2\pi \int_a^b x\left[(-x^4 + 4x - 1) - (x^3 - x + 1)\right] dx$$
$$= 2\pi \int_a^b x(-x^4 - x^3 + 5x - 2)\, dx \approx 3.17$$

35. $V = 2\pi \displaystyle\int_0^{\pi/2} \left[\left(\tfrac{\pi}{2} - x\right)\left(\sin^2 x - \sin^4 x\right)\right] dx$

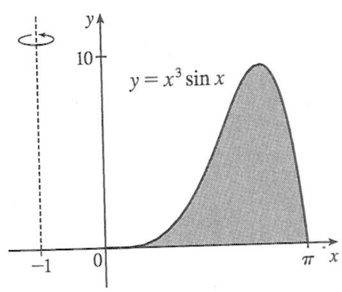
$\overset{\text{CAS}}{=} \frac{1}{32}\pi^3$



36. $V = 2\pi \int_0^{\pi} \{[x - (-1)](x^3 \sin x)\}\, dx$

$\overset{\text{CAS}}{=} 2\pi(\pi^4 + \pi^3 - 12\pi^2 - 6\pi + 48)$

$= 2\pi^5 + 2\pi^4 - 24\pi^3 - 12\pi^2 + 96\pi$



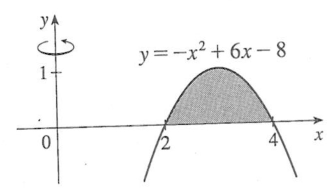
37. Use shells:

$V = \int_2^4 2\pi x(-x^2 + 6x - 8)\, dx = 2\pi \int_2^4 (-x^3 + 6x^2 - 8x)\, dx$

$= 2\pi\left[-\frac{1}{4}x^4 + 2x^3 - 4x^2\right]_2^4$

$= 2\pi[(-64 + 128 - 64) - (-4 + 16 - 16)]$

$= 2\pi(4) = 8\pi$



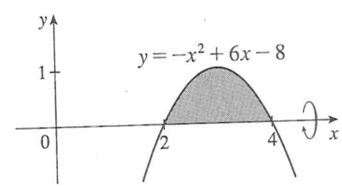
38. Use disks:

$V = \int_2^4 \pi(-x^2 + 6x - 8)^2\, dx$

$= \pi \int_2^4 (x^4 - 12x^3 + 52x^2 - 96x + 64)\, dx$

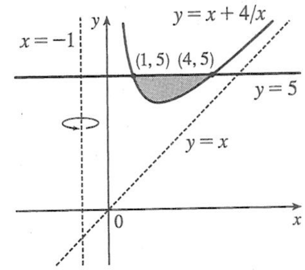
$= \pi\left[\frac{1}{5}x^5 - 3x^4 + \frac{52}{3}x^3 - 48x^2 + 64x\right]_2^4$

$= \pi\left(\frac{512}{15} - \frac{496}{15}\right) = \frac{16}{15}\pi$



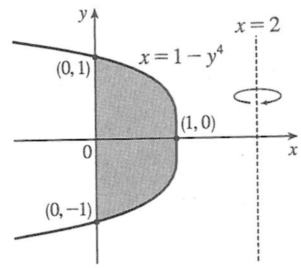
39. Use shells:

$$V = \int_1^4 2\pi[x-(-1)][5-(x+4/x)]\,dx$$
$$= 2\pi \int_1^4 (x+1)(5-x-4/x)\,dx$$
$$= 2\pi \int_1^4 (5x - x^2 - 4 + 5 - x - 4/x)\,dx$$
$$= 2\pi \int_1^4 (-x^2+4x+1-4/x)\,dx = 2\pi\left[-\tfrac{1}{3}x^3 + 2x^2 + x - 4\ln x\right]_1^4$$
$$= 2\pi\left[\left(-\tfrac{64}{3} + 32 + 4 - 4\ln 4\right) - \left(-\tfrac{1}{3} + 2 + 1 - 0\right)\right]$$
$$= 2\pi(12 - 4\ln 4) = 8\pi(3 - \ln 4)$$



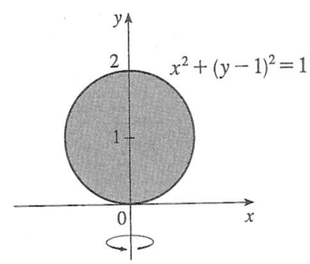
40. Use washers:

$$V = \int_{-1}^1 \pi\left\{[2-0]^2 - [2-(1-y^4)]^2\right\} dy$$
$$= 2\pi \int_0^1 \left[4 - (1+y^4)^2\right] dy \qquad \text{[by symmetry]}$$
$$= 2\pi \int_0^1 \left[4 - (1 + 2y^4 + y^8)\right] dy = 2\pi \int_0^1 (3 - 2y^4 - y^8)\,dy$$
$$= 2\pi\left[3y - \tfrac{2}{5}y^5 - \tfrac{1}{9}y^9\right]_0^1 = 2\pi\left(3 - \tfrac{2}{5} - \tfrac{1}{9}\right)$$
$$= 2\pi\left(\tfrac{112}{45}\right) = \tfrac{224}{45}\pi$$



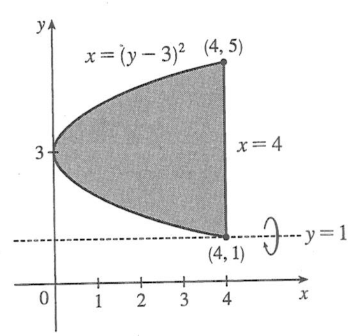
41. Use disks: $x^2 + (y-1)^2 = 1 \;\Leftrightarrow\; x = \pm\sqrt{1-(y-1)^2}$

$$V = \pi \int_0^2 \left[\sqrt{1-(y-1)^2}\right]^2 dy = \pi \int_0^2 (2y - y^2)\,dy$$
$$= \pi\left[y^2 - \tfrac{1}{3}y^3\right]_0^2 = \pi\left(4 - \tfrac{8}{3}\right) = \tfrac{4}{3}\pi$$



42. Use shells:

$$V = \int_1^5 2\pi(y-1)[4-(y-3)^2]\,dy$$
$$= 2\pi \int_1^5 (y-1)(-y^2+6y-5)\,dy$$
$$= 2\pi \int_1^5 (-y^3 + 7y^2 - 11y + 5)\,dy$$
$$= 2\pi\left[-\tfrac{1}{4}y^4 + \tfrac{7}{3}y^3 - \tfrac{11}{2}y^2 + 5y\right]_1^5$$
$$= 2\pi\left(\tfrac{275}{12} - \tfrac{19}{12}\right) = \tfrac{128}{3}\pi$$



43. Use shells:

$$V = 2\int_0^r 2\pi x\sqrt{r^2-x^2}\,dx$$
$$= -2\pi \int_0^r (r^2-x^2)^{1/2}(-2x)\,dx$$
$$= \left[-2\pi \cdot \tfrac{2}{3}(r^2-x^2)^{3/2}\right]_0^r$$
$$= -\tfrac{4}{3}\pi(0 - r^3) = \tfrac{4}{3}\pi r^3$$