

With the second term ($n = 1$), $\frac{1}{\pi} \approx \frac{2\sqrt{2}}{9801} \left(\frac{1103}{1} + \frac{4!(1103 + 26,390)}{396^4} \right) \Rightarrow \pi \approx 3.141\,592\,653\,589\,793\,878$, so we get 15 correct decimal places of π .

39. (a) Since $\sum a_n$ is absolutely convergent, and since $|a_n^+| \leq |a_n|$ and $|a_n^-| \leq |a_n|$ (because a_n^+ and a_n^- each equal either a_n or 0), we conclude by the Comparison Test that both $\sum a_n^+$ and $\sum a_n^-$ must be absolutely convergent.
Or: Use Theorem 11.2.8.
- (b) We will show by contradiction that both $\sum a_n^+$ and $\sum a_n^-$ must diverge. For suppose that $\sum a_n^+$ converged. Then so would $\sum (a_n^+ - \frac{1}{2}a_n)$ by Theorem 11.2.8. But $\sum (a_n^+ - \frac{1}{2}a_n) = \sum [\frac{1}{2}(a_n + |a_n|) - \frac{1}{2}a_n] = \frac{1}{2} \sum |a_n|$, which diverges because $\sum a_n$ is only conditionally convergent. Hence, $\sum a_n^+$ can't converge. Similarly, neither can $\sum a_n^-$.
40. Let $\sum b_n$ be the rearranged series constructed in the hint. [This series can be constructed by virtue of the result of Exercise 39(b).] This series will have partial sums s_n that oscillate in value back and forth across r . Since $\lim_{n \rightarrow \infty} a_n = 0$ (by Theorem 11.2.6), and since the size of the oscillations $|s_n - r|$ is always less than $|a_n|$ because of the way $\sum b_n$ was constructed, we have that $\sum b_n = \lim_{n \rightarrow \infty} s_n = r$.

11.7 Strategy for Testing Series

1. $\frac{1}{n+3^n} < \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$ for all $n \geq 1$. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ is a convergent geometric series [$|r| = \frac{1}{3} < 1$], so $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$ converges by the Comparison Test.
2. $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(2n+1)^n}{n^{2n}} \right|} = \lim_{n \rightarrow \infty} \frac{2n+1}{n^2} = \lim_{n \rightarrow \infty} \left(\frac{2}{n} + \frac{1}{n^2} \right) = 0 < 1$, so the series $\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$ converges by the Root Test.
3. $\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{n}{n+2} = 1$, so $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (-1)^n \frac{n}{n+2}$ does not exist. Thus, the series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$ diverges by the Test for Divergence.
4. $b_n = \frac{n}{n^2+2} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since $\left(\frac{x}{x^2+2}\right)' = \frac{(x^2+2)(1) - x(2x)}{(x^2+2)^2} = \frac{2-x^2}{(x^2+2)^2} < 0$ for $x \geq \sqrt{2}$. Also, $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{n}{n^2+2} = \lim_{n \rightarrow \infty} \frac{1/n}{1+2/n^2} = 0$. Thus, the series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+2}$ converges by the Alternating Series Test.
5. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)^2 2^n}{(-5)^{n+1}} \cdot \frac{(-5)^n}{n^2 2^{n-1}} \right| = \lim_{n \rightarrow \infty} \frac{2(n+1)^2}{5n^2} = \frac{2}{5} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^2 = \frac{2}{5}(1) = \frac{2}{5} < 1$, so the series $\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$ converges by the Ratio Test.

6. Use the Limit Comparison Test with $a_n = \frac{1}{2n+1}$ and $b_n = \frac{1}{n}$: $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n}{2n+1} = \lim_{n \rightarrow \infty} \frac{1}{2+(1/n)} = \frac{1}{2} > 0$.

Since the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n=1}^{\infty} \frac{1}{2n+1}$. [Or: Use the Integral Test.]

7. Let $f(x) = \frac{1}{x\sqrt{\ln x}}$. Then f is positive, continuous, and decreasing on $[2, \infty)$, so we can apply the Integral Test.

Since $\int \frac{1}{x\sqrt{\ln x}} dx \left[\begin{array}{l} u = \ln x, \\ du = dx/x \end{array} \right] = \int u^{-1/2} du = 2u^{1/2} + C = 2\sqrt{\ln x} + C$, we find

$\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}} = \lim_{t \rightarrow \infty} \int_2^t \frac{dx}{x\sqrt{\ln x}} = \lim_{t \rightarrow \infty} [2\sqrt{\ln x}]_2^t = \lim_{t \rightarrow \infty} (2\sqrt{\ln t} - 2\sqrt{\ln 2}) = \infty$. Since the integral diverges, the

given series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ diverges.

8. $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!} = \sum_{k=1}^{\infty} \frac{2^k}{(k+1)(k+2)}$. Using the Ratio Test, we get

$\lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \rightarrow \infty} \left| \frac{2^{k+1}}{(k+2)(k+3)} \cdot \frac{(k+1)(k+2)}{2^k} \right| = \lim_{k \rightarrow \infty} \left(2 \cdot \frac{k+1}{k+3} \right) = 2 > 1$, so the series diverges.

Or: Use the Test for Divergence.

9. $\sum_{k=1}^{\infty} k^2 e^{-k} = \sum_{k=1}^{\infty} \frac{k^2}{e^k}$. Using the Ratio Test, we get

$\lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \rightarrow \infty} \left| \frac{(k+1)^2}{e^{k+1}} \cdot \frac{e^k}{k^2} \right| = \lim_{k \rightarrow \infty} \left[\left(\frac{k+1}{k} \right)^2 \cdot \frac{1}{e} \right] = 1^2 \cdot \frac{1}{e} = \frac{1}{e} < 1$, so the series converges.

10. Let $f(x) = x^2 e^{-x^3}$. Then f is continuous and positive on $[1, \infty)$, and $f'(x) = \frac{x(2-3x^3)}{e^{x^3}} < 0$ for $x \geq 1$, so f is

decreasing on $[1, \infty)$ as well, and we can apply the Integral Test. $\int_1^{\infty} x^2 e^{-x^3} dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{3} e^{-x^3} \right]_1^t = \frac{1}{3e}$, so the integral converges, and hence, the series converges.

11. $b_n = \frac{1}{n \ln n} > 0$ for $n \geq 2$, $\{b_n\}$ is decreasing, and $\lim_{n \rightarrow \infty} b_n = 0$, so the given series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$ converges by the Alternating Series Test.

12. The series $\sum_{n=1}^{\infty} \sin n$ diverges by the Test for Divergence since $\lim_{n \rightarrow \infty} \sin n$ does not exist.

13. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{3^{n+1}(n+1)^2}{(n+1)!} \cdot \frac{n!}{3^n n^2} \right| = \lim_{n \rightarrow \infty} \frac{3(n+1)^2}{(n+1)n^2} = 3 \lim_{n \rightarrow \infty} \frac{n+1}{n^2} = 0 < 1$, so the series $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ converges by the Ratio Test.

14. $\left| \frac{\sin 2n}{1+2^n} \right| \leq \frac{1}{1+2^n} < \frac{1}{2^n} = \left(\frac{1}{2} \right)^n$, so the series $\sum_{n=1}^{\infty} \left| \frac{\sin 2n}{1+2^n} \right|$ converges by comparison with the geometric series $\sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n$ with $|r| = \frac{1}{2} < 1$. Thus, the series $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$ converges absolutely, implying convergence.

$$15. \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)!}{2 \cdot 5 \cdot 8 \cdots (3n+2)[3(n+1)+2]} \cdot \frac{2 \cdot 5 \cdot 8 \cdots (3n+2)}{n!} \right| = \lim_{n \rightarrow \infty} \frac{n+1}{3n+5} = \frac{1}{3} < 1,$$

so the series $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$ converges by the Ratio Test.

16. Using the Limit Comparison Test with $a_n = \frac{n^2+1}{n^3+1}$ and $b_n = \frac{1}{n}$, we have

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \left(\frac{n^2+1}{n^3+1} \cdot \frac{n}{1} \right) = \lim_{n \rightarrow \infty} \frac{n^3+n}{n^3+1} = \lim_{n \rightarrow \infty} \frac{1+1/n^2}{1+1/n^3} = 1 > 0. \text{ Since } \sum_{n=1}^{\infty} b_n \text{ is the divergent harmonic series, } \sum_{n=1}^{\infty} a_n \text{ is also divergent.}$$

17. $\lim_{n \rightarrow \infty} 2^{1/n} = 2^0 = 1$, so $\lim_{n \rightarrow \infty} (-1)^n 2^{1/n}$ does not exist and the series $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$ diverges by the Test for Divergence.

18. $b_n = \frac{1}{\sqrt{n}-1}$ for $n \geq 2$. $\{b_n\}$ is a decreasing sequence of positive numbers and $\lim_{n \rightarrow \infty} b_n = 0$, so $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$ converges by the Alternating Series Test.

19. Let $f(x) = \frac{\ln x}{\sqrt{x}}$. Then $f'(x) = \frac{2 - \ln x}{2x^{3/2}} < 0$ when $\ln x > 2$ or $x > e^2$, so $\frac{\ln n}{\sqrt{n}}$ is decreasing for $n > e^2$.

By l'Hospital's Rule, $\lim_{n \rightarrow \infty} \frac{\ln n}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1/n}{1/(2\sqrt{n})} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n}} = 0$, so the series $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$ converges by the

Alternating Series Test.

20. $\lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \rightarrow \infty} \left| \frac{k+6}{5^{k+1}} \cdot \frac{5^k}{k+5} \right| = \frac{1}{5} \lim_{k \rightarrow \infty} \frac{k+6}{k+5} = \frac{1}{5} < 1$, so the series $\sum_{k=1}^{\infty} \frac{k+5}{5^k}$ converges by the Ratio Test.

21. $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n} = \sum_{n=1}^{\infty} \left(\frac{4}{n} \right)^n$. $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \frac{4}{n} = 0 < 1$, so the given series is absolutely convergent by the Root Test.

22. $\frac{\sqrt{n^2-1}}{n^3+2n^2+5} < \frac{n}{n^3+2n^2+5} < \frac{n}{n^3} = \frac{1}{n^2}$ for $n \geq 1$, so $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ converges by the Comparison Test with the convergent p -series $\sum_{n=1}^{\infty} 1/n^2$ [$p = 2 > 1$].

23. Using the Limit Comparison Test with $a_n = \tan\left(\frac{1}{n}\right)$ and $b_n = \frac{1}{n}$, we have

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\tan(1/n)}{1/n} = \lim_{x \rightarrow \infty} \frac{\tan(1/x)}{1/x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\sec^2(1/x) \cdot (-1/x^2)}{-1/x^2} = \lim_{x \rightarrow \infty} \sec^2(1/x) = 1^2 = 1 > 0. \text{ Since}$$

$\sum_{n=1}^{\infty} b_n$ is the divergent harmonic series, $\sum_{n=1}^{\infty} a_n$ is also divergent.

24. $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(n \sin \frac{1}{n} \right) = \lim_{n \rightarrow \infty} \frac{\sin(1/n)}{1/n} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1 \neq 0$, so the series $\sum_{n=1}^{\infty} n \sin(1/n)$ diverges by the Test for Divergence.

25. Use the Ratio Test. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)! \cdot e^{n^2}}{e^{(n+1)^2} \cdot n!} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)n! \cdot e^{n^2}}{e^{n^2+2n+1}n!} = \lim_{n \rightarrow \infty} \frac{n+1}{e^{2n+1}} = 0 < 1$, so $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

converges.

26. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \left(\frac{n^2 + 2n + 2}{5^{n+1}} \cdot \frac{5^n}{n^2 + 1} \right) = \lim_{n \rightarrow \infty} \left(\frac{1 + 2/n + 2/n^2}{1 + 1/n^2} \cdot \frac{1}{5} \right) = \frac{1}{5} < 1$, so $\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$

converges by the Ratio Test.

27. $\int_2^{\infty} \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_1^t$ [using integration by parts] $\stackrel{H}{=} 1$. So $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ converges by the Integral Test, and since $\frac{k \ln k}{(k+1)^3} < \frac{k \ln k}{k^3} = \frac{\ln k}{k^2}$, the given series $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$ converges by the Comparison Test.

28. Since $\left\{ \frac{1}{n} \right\}$ is a decreasing sequence, $e^{1/n} \leq e^{1/1} = e$ for all $n \geq 1$, and $\sum_{n=1}^{\infty} \frac{e}{n^2}$ converges ($p = 2 > 1$), so $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges by the Comparison Test. (Or use the Integral Test.)

29. $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\cosh n} = \sum_{n=1}^{\infty} (-1)^n b_n$. Now $b_n = \frac{1}{\cosh n} > 0$, $\{b_n\}$ is decreasing, and $\lim_{n \rightarrow \infty} b_n = 0$, so the series converges by the Alternating Series Test.

Or: Write $\frac{1}{\cosh n} = \frac{2}{e^n + e^{-n}} < \frac{2}{e^n}$ and $\sum_{n=1}^{\infty} \frac{1}{e^n}$ is a convergent geometric series, so $\sum_{n=1}^{\infty} \frac{1}{\cosh n}$ is convergent by the

Comparison Test. So $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\cosh n}$ is absolutely convergent and therefore convergent.

30. Let $f(x) = \frac{\sqrt{x}}{x+5}$. Then $f(x)$ is continuous and positive on $[1, \infty)$, and since $f'(x) = \frac{5-x}{2\sqrt{x}(x+5)^2} < 0$ for $x > 5$, $f(x)$ is

eventually decreasing, so we can use the Alternating Series Test. $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n+5} = \lim_{n \rightarrow \infty} \frac{1}{n^{1/2} + 5n^{-1/2}} = 0$, so the series

$$\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5} \text{ converges.}$$

31. $\lim_{k \rightarrow \infty} a_k = \lim_{k \rightarrow \infty} \frac{5^k}{3^k + 4^k} = [\text{divide by } 4^k] \lim_{k \rightarrow \infty} \frac{(5/4)^k}{(3/4)^k + 1} = \infty$ since $\lim_{k \rightarrow \infty} \left(\frac{3}{4}\right)^k = 0$ and $\lim_{k \rightarrow \infty} \left(\frac{5}{4}\right)^k = \infty$.

Thus, $\sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}$ diverges by the Test for Divergence.

32. $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(n!)^n}{n^{4n}} \right|} = \lim_{n \rightarrow \infty} \frac{n!}{n^4} = \lim_{n \rightarrow \infty} \left[\frac{n}{n} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n} \cdot \frac{n-3}{n} \cdot (n-4)! \right]$

$$= \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \left(1 - \frac{3}{n}\right) (n-4)! \right] = \infty,$$

so the series $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$ diverges by the Root Test.

33. Let $a_n = \frac{\sin(1/n)}{\sqrt{n}}$ and $b_n = \frac{1}{n\sqrt{n}}$. Then $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\sin(1/n)}{1/n} = 1 > 0$, so $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$ converges by limit

comparison with the convergent p -series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ [$p = 3/2 > 1$].

34. $0 \leq n \cos^2 n \leq n$, so $\frac{1}{n + n \cos^2 n} \geq \frac{1}{n + n} = \frac{1}{2n}$. Thus, $\sum_{n=1}^{\infty} \frac{1}{n + n \cos^2 n}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{2n}$, which is a constant multiple of the (divergent) harmonic series.

35. $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^{n^2/n} = \lim_{n \rightarrow \infty} \frac{1}{[(n+1)/n]^n} = \frac{1}{\lim_{n \rightarrow \infty} (1 + 1/n)^n} = \frac{1}{e} < 1$, so the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$ converges by the Root Test.

36. Note that $(\ln n)^{\ln n} = (e^{\ln \ln n})^{\ln n} = (e^{\ln n})^{\ln \ln n} = n^{\ln \ln n}$ and $\ln \ln n \rightarrow \infty$ as $n \rightarrow \infty$, so $\ln \ln n > 2$ for sufficiently large n . For these n we have $(\ln n)^{\ln n} > n^2$, so $\frac{1}{(\ln n)^{\ln n}} < \frac{1}{n^2}$. Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges [$p = 2 > 1$], so does $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ by the Comparison Test.

37. $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} (2^{1/n} - 1) = 1 - 1 = 0 < 1$, so the series $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$ converges by the Root Test.

38. Use the Limit Comparison Test with $a_n = \sqrt[n]{2} - 1$ and $b_n = 1/n$. Then

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{2^{1/n} - 1}{1/n} = \lim_{x \rightarrow \infty} \frac{2^{1/x} - 1}{1/x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2^{1/x} \cdot \ln 2 \cdot (-1/x^2)}{-1/x^2} = \lim_{x \rightarrow \infty} (2^{1/x} \cdot \ln 2) = 1 \cdot \ln 2 = \ln 2 > 0.$$

So since $\sum_{n=1}^{\infty} b_n$ diverges (harmonic series), so does $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$.

Alternate solution: $\sqrt[n]{2} - 1 = \frac{1}{2^{(n-1)/n} + 2^{(n-2)/n} + 2^{(n-3)/n} + \dots + 2^{1/n} + 1}$ [rationalize the numerator] $\geq \frac{1}{2n}$,

and since $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series), so does $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$ by the Comparison Test.

11.8 Power Series

1. A power series is a series of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$, where x is a variable and the c_n 's are constants called the coefficients of the series.

More generally, a series of the form $\sum_{n=0}^{\infty} c_n (x - a)^n = c_0 + c_1 (x - a) + c_2 (x - a)^2 + \dots$ is called a power series in $(x - a)$ or a power series centered at a or a power series about a , where a is a constant.

2. (a) Given the power series $\sum_{n=0}^{\infty} c_n (x - a)^n$, the radius of convergence is:

(i) 0 if the series converges only when $x = a$

(ii) ∞ if the series converges for all x , or

(iii) a positive number R such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$.

In most cases, R can be found by using the Ratio Test.

(b) The interval of convergence of a power series is the interval that consists of all values of x for which the series converges.

Corresponding to the cases in part (a), the interval of convergence is: (i) the single point $\{a\}$, (ii) all real numbers; that is, the real number line $(-\infty, \infty)$, or (iii) an interval with endpoints $a - R$ and $a + R$ which can contain neither, either, or both of the endpoints. In this case, we must test the series for convergence at each endpoint to determine the interval of convergence.