Calculus (ET), Stewart, 6th ed.

11.4 EXERCISES

1. Suppose Σa_n and Σb_n are series with positive terms and Σb_n is known to be convergent.
   (a) If $a_n > b_n$ for all n, what can you say about Σa_n? Why?
   (b) If $a_n < b_n$ for all n, what can you say about Σa_n? Why?

2. Suppose Σa_n and Σb_n are series with positive terms and Σb_n is known to be divergent.
   (a) If $a_n > b_n$ for all n, what can you say about Σa_n? Why?
   (b) If $a_n < b_n$ for all n, what can you say about Σa_n? Why?

3–32 Determine whether the series converges or diverges.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

12. $\displaystyle\sum_{n=0}^{\infty} \frac{1+\sin n}{10^n}$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^{1.2}}$

14. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n+3}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$

22. $\displaystyle\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{1+n+n^2}{\sqrt{1+n^2+n^6}}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$

27. $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

31. $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

33–36 Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.

33. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3}$

35. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+2^n}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)3^n}$

37. The meaning of the decimal representation of a number $0.d_1d_2d_3\ldots$ (where the digit d_i is one of the numbers 0, 1, 2, ..., 9) is that

$$0.d_1d_2d_3d_4\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \frac{d_4}{10^4} + \cdots$$

Show that this series always converges.

38. For what values of p does the series $\Sigma_{n=2}^{\infty} 1/(n^p \ln n)$ converge?

39. Prove that if $a_n \geqslant 0$ and Σa_n converges, then Σa_n^2 also converges.

40. (a) Suppose that Σa_n and Σb_n are series with positive terms and Σb_n is convergent. Prove that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$

then Σa_n is also convergent.
(b) Use part (a) to show that the series converges.
(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}\, e^n}$

41. (a) Suppose that Σa_n and Σb_n are series with positive terms and Σb_n is divergent. Prove that if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$$

then Σa_n is also divergent.
(b) Use part (a) to show that the series diverges.
(i) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

42. Give an example of a pair of series Σa_n and Σb_n with positive terms where $\lim_{n\to\infty}(a_n/b_n) = 0$ and Σb_n diverges, but Σa_n converges. (Compare with Exercise 40.)

43. Show that if $a_n > 0$ and $\lim_{n\to\infty} na_n \neq 0$, then Σa_n is divergent.

44. Show that if $a_n > 0$ and Σa_n is convergent, then $\Sigma \ln(1+a_n)$ is convergent.

45. If Σa_n is a convergent series with positive terms, is it true that $\Sigma \sin(a_n)$ is also convergent?

46. If Σa_n and Σb_n are both convergent series with positive terms, is it true that Σa_nb_n is also convergent?