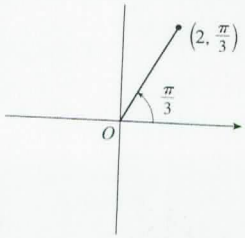
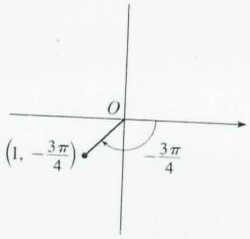
 Stewart ET 6th ed. p.826

1. (a) $\left(2, \frac{\pi}{3}\right)$



By adding 2π to $\frac{\pi}{3}$, we obtain the point $\left(2, \frac{7\pi}{3}\right)$. The direction opposite $\frac{\pi}{3}$ is $\frac{4\pi}{3}$, so $\left(-2, \frac{4\pi}{3}\right)$ is a point that satisfies the $r < 0$ requirement.

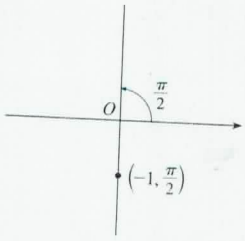
(b) $\left(1, -\frac{3\pi}{4}\right)$



$r > 0$: $\left(1, -\frac{3\pi}{4} + 2\pi\right) = \left(1, \frac{5\pi}{4}\right)$

$r < 0$: $\left(-1, -\frac{3\pi}{4} + \pi\right) = \left(-1, \frac{\pi}{4}\right)$

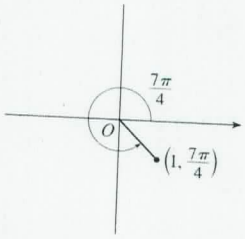
(c) $\left(-1, \frac{\pi}{2}\right)$



$r > 0$: $\left(-(-1), \frac{\pi}{2} + \pi\right) = \left(1, \frac{3\pi}{2}\right)$

$r < 0$: $\left(-1, \frac{\pi}{2} + 2\pi\right) = \left(-1, \frac{5\pi}{2}\right)$

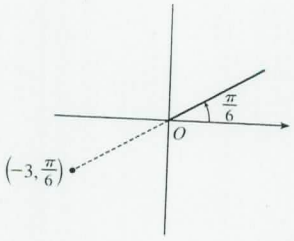
2. (a) $\left(1, \frac{7\pi}{4}\right)$



$r > 0$: $\left(1, \frac{7\pi}{4} - 2\pi\right) = \left(1, -\frac{\pi}{4}\right)$

$r < 0$: $\left(-1, \frac{7\pi}{4} - \pi\right) = \left(-1, \frac{3\pi}{4}\right)$

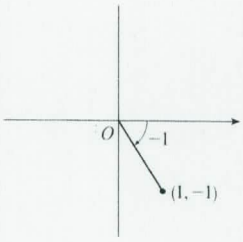
(b) $\left(-3, \frac{\pi}{6}\right)$



$r > 0$: $\left(-(-3), \frac{\pi}{6} + \pi\right) = \left(3, \frac{7\pi}{6}\right)$

$r < 0$: $\left(-3, \frac{\pi}{6} + 2\pi\right) = \left(-3, \frac{13\pi}{6}\right)$

(c) $(1, -1)$

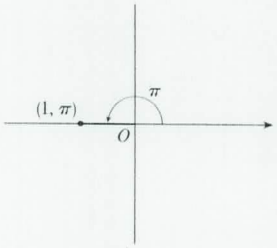


$\theta = -1$ radian $\approx -57.3°$

$\boldsymbol{r > 0}$**:** $(1, -1 + 2\pi)$

$\boldsymbol{r < 0}$**:** $(-1, -1 + \pi)$

3. (a)

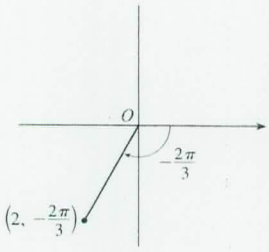


$x = 1\cos\pi = 1(-1) = -1$ and

$y = 1\sin\pi = 1(0) = 0$ give us

the Cartesian coordinates $(-1, 0)$.

(b)

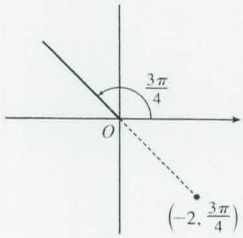


$x = 2\cos\left(-\frac{2\pi}{3}\right) = 2\left(-\frac{1}{2}\right) = -1$ and

$y = 2\sin\left(-\frac{2\pi}{3}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

give us $\left(-1, -\sqrt{3}\right)$.

(c)

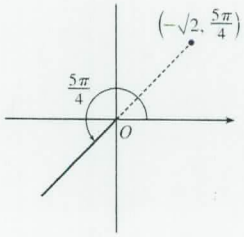


$x = -2\cos\frac{3\pi}{4} = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$ and

$y = -2\sin\frac{3\pi}{4} = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

gives us $\left(\sqrt{2}, -\sqrt{2}\right)$.

4. (a)

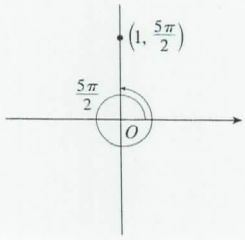


$x = -\sqrt{2}\cos\frac{5\pi}{4} = -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = 1$ and

$y = -\sqrt{2}\sin\frac{5\pi}{4} = -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = 1$

gives us $(1, 1)$.

(b)

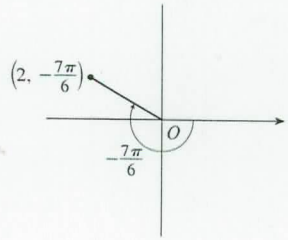


$x = 1\cos\frac{5\pi}{2} = 1(0) = 0$ and

$y = 1\sin\frac{5\pi}{2} = 1(1) = 1$

gives us $(0, 1)$.

(c)



$x = 2\cos\left(-\frac{7\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$ and

$y = 2\sin\left(-\frac{7\pi}{6}\right) = 2\left(\frac{1}{2}\right) = 1$

give us $\left(-\sqrt{3}, 1\right)$.

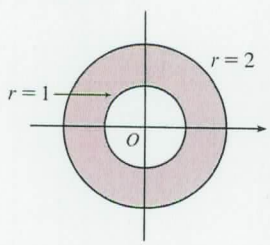
5. (a) $x = 2$ and $y = -2 \;\Rightarrow\; r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$ and $\theta = \tan^{-1}\left(\frac{-2}{2}\right) = -\frac{\pi}{4}$. Since $(2, -2)$ is in the fourth quadrant, the polar coordinates are (i) $\left(2\sqrt{2}, \frac{7\pi}{4}\right)$ and (ii) $\left(-2\sqrt{2}, \frac{3\pi}{4}\right)$.

(b) $x = -1$ and $y = \sqrt{3} \;\Rightarrow\; r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = 2$ and $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = \frac{2\pi}{3}$. Since $\left(-1, \sqrt{3}\right)$ is in the second quadrant, the polar coordinates are (i) $\left(2, \frac{2\pi}{3}\right)$ and (ii) $\left(-2, \frac{5\pi}{3}\right)$.

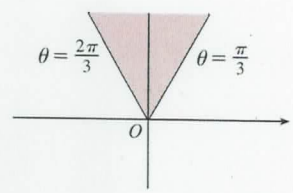
6. (a) $x = 3\sqrt{3}$ and $y = 3 \;\Rightarrow\; r = \sqrt{\left(3\sqrt{3}\right)^2 + 3^2} = \sqrt{27 + 9} = 6$ and $\theta = \tan^{-1}\left(\frac{3}{3\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$. Since $\left(3\sqrt{3}, 3\right)$ is in the first quadrant, the polar coordinates are (i) $\left(6, \frac{\pi}{6}\right)$ and (ii) $\left(-6, \frac{7\pi}{6}\right)$.

(b) $x = 1$ and $y = -2 \;\Rightarrow\; r = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ and $\theta = \tan^{-1}\left(\frac{-2}{1}\right) = -\tan^{-1} 2$. Since $(1, -2)$ is in the fourth quadrant, the polar coordinates are (i) $\left(\sqrt{5}, 2\pi - \tan^{-1} 2\right)$ and (ii) $\left(-\sqrt{5}, \pi - \tan^{-1} 2\right)$.

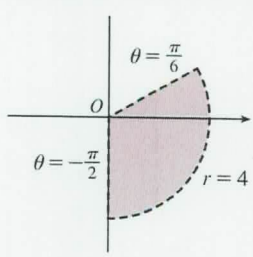
7. The curves $r = 1$ and $r = 2$ represent circles with center O and radii 1 and 2. The region in the plane satisfying $1 \le r \le 2$ consists of both circles and the shaded region between them in the figure.



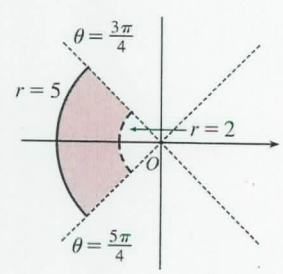
8. $r \ge 0, \quad \pi/3 \le \theta \le 2\pi/3$



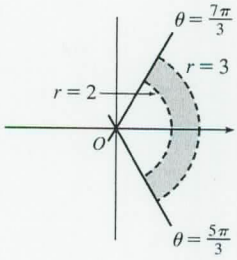
9. The region satisfying $0 \le r < 4$ and $-\pi/2 \le \theta < \pi/6$ does not include the circle $r = 4$ nor the line $\theta = \frac{\pi}{6}$.



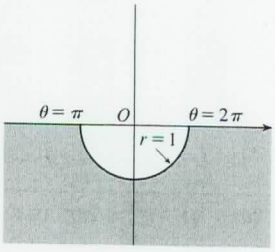
10. $2 < r \le 5, \; 3\pi/4 < \theta < 5\pi/4$



11. $2 < r < 3, \quad \frac{5\pi}{3} \le \theta \le \frac{7\pi}{3}$



12. $r \ge 1, \pi \le \theta \le 2\pi$



13. Converting the polar coordinates $(2, \pi/3)$ and $(4, 2\pi/3)$ to Cartesian coordinates gives us $\left(2\cos\frac{\pi}{3}, 2\sin\frac{\pi}{3}\right) = \left(1, \sqrt{3}\right)$ and $\left(4\cos\frac{2\pi}{3}, 4\sin\frac{2\pi}{3}\right) = \left(-2, 2\sqrt{3}\right)$. Now use the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2-1)^2 + \left(2\sqrt{3} - \sqrt{3}\right)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$$

14. The points (r_1, θ_1) and (r_2, θ_2) in Cartesian coordinates are $(r_1\cos\theta_1, r_1\sin\theta_1)$ and $(r_2\cos\theta_2, r_2\sin\theta_2)$, respectively. The *square* of the distance between them is

$$\begin{aligned}
&(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2 \\
&\qquad = \left(r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_1^2\cos^2\theta_1\right) + \left(r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_1^2\sin^2\theta_1\right) \\
&\qquad = r_1^2\left(\sin^2\theta_1 + \cos^2\theta_1\right) + r_2^2\left(\sin^2\theta_2 + \cos^2\theta_2\right) - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \\
&\qquad = r_1^2 - 2r_1r_2\cos(\theta_1 - \theta_2) + r_2^2,
\end{aligned}$$

so the distance between them is $\sqrt{r_1^2 - 2r_1r_2\cos(\theta_1 - \theta_2) + r_2^2}$.

15. $r = 2 \quad \Leftrightarrow \quad \sqrt{x^2 + y^2} = 2 \quad \Leftrightarrow \quad x^2 + y^2 = 4$, a circle of radius 2 centered at the origin.

16. $r\cos\theta = 1 \quad \Leftrightarrow \quad x = 1$, a vertical line.

17. $r = 3\sin\theta \quad \Rightarrow \quad r^2 = 3r\sin\theta \quad \Leftrightarrow \quad x^2 + y^2 = 3y \quad \Leftrightarrow \quad x^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$, a circle of radius $\frac{3}{2}$ centered at $\left(0, \frac{3}{2}\right)$. The first two equations are actually equivalent since $r^2 = 3r\sin\theta \quad \Rightarrow \quad r(r - 3\sin\theta) = 0 \quad \Rightarrow \quad r = 0$ or $r = 3\sin\theta$. But $r = 3\sin\theta$ gives the point $r = 0$ (the pole) when $\theta = 0$. Thus, the single equation $r = 3\sin\theta$ is equivalent to the compound condition ($r = 0$ or $r = 3\sin\theta$).

18. $r = 2\sin\theta + 2\cos\theta \quad \Rightarrow \quad r^2 = 2r\sin\theta + 2r\cos\theta \quad \Leftrightarrow \quad x^2 + y^2 = 2y + 2x \quad \Leftrightarrow$ $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 2 \quad \Leftrightarrow \quad (x-1)^2 + (y-1)^2 = 2$. The first implication is reversible since $r^2 = 2r\sin\theta + 2r\cos\theta \quad \Rightarrow \quad r = 0$ or $r = 2\sin\theta + 2\cos\theta$, but the curve $r = 2\sin\theta + 2\cos\theta$ passes through the pole ($r = 0$) when $\theta = -\frac{\pi}{4}$, so $r = 2\sin\theta + 2\cos\theta$ includes the single point of $r = 0$. The curve is a circle of radius $\sqrt{2}$, centered at $(1, 1)$.

19. $r = \csc\theta \quad \Leftrightarrow \quad r = \dfrac{1}{\sin\theta} \quad \Leftrightarrow \quad r\sin\theta = 1 \quad \Leftrightarrow \quad y = 1$, a horizontal line 1 unit above the x-axis.

20. $r = \tan\theta\sec\theta = \dfrac{\sin\theta}{\cos^2\theta} \quad \Rightarrow \quad r\cos^2\theta = \sin\theta \quad \Leftrightarrow \quad (r\cos\theta)^2 = r\sin\theta \quad \Leftrightarrow \quad x^2 = y$, a parabola with vertex at the origin opening upward. The first implication is reversible since $\cos\theta = 0$ would imply $\sin\theta = r\cos^2\theta = 0$, contradicting the fact that $\cos^2\theta + \sin^2\theta = 1$.

21. $x = 3 \iff r\cos\theta = 3 \iff r = 3/\cos\theta \iff r = 3\sec\theta$.

22. $x^2 + y^2 = 9 \iff r^2 = 9 \iff r = 3$. [$r = -3$ gives the same curve.]

23. $x = -y^2 \iff r\cos\theta = -r^2\sin^2\theta \iff \cos\theta = -r\sin^2\theta \iff r = -\dfrac{\cos\theta}{\sin^2\theta} = -\cot\theta\csc\theta$.

24. $x + y = 9 \iff r\cos\theta + r\sin\theta = 9 \iff r = 9/(\cos\theta + \sin\theta)$.

25. $x^2 + y^2 = 2cx \iff r^2 = 2cr\cos\theta \iff r^2 - 2cr\cos\theta = 0 \iff r(r - 2c\cos\theta) = 0 \iff r = 0$ or $r = 2c\cos\theta$. $r = 0$ is included in $r = 2c\cos\theta$ when $\theta = \frac{\pi}{2} + n\pi$, so the curve is represented by the single equation $r = 2c\cos\theta$.

26. $xy = 4 \iff (r\cos\theta)(r\sin\theta) = 4 \iff r^2\left(\frac{1}{2}\cdot 2\sin\theta\cos\theta\right) = 4 \iff r^2\sin 2\theta = 8 \implies r^2 = 8\csc 2\theta$

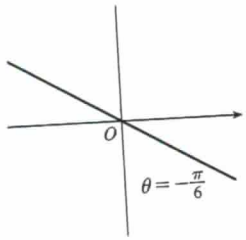
27. (a) The description leads immediately to the polar equation $\theta = \frac{\pi}{6}$, and the Cartesian equation $y = \tan\left(\frac{\pi}{6}\right)x = \frac{1}{\sqrt{3}}x$ is slightly more difficult to derive.

(b) The easier description here is the Cartesian equation $x = 3$.

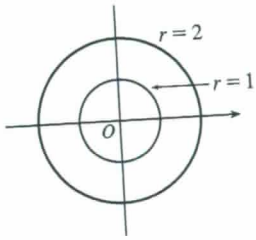
28. (a) Because its center is not at the origin, it is more easily described by its Cartesian equation, $(x-2)^2 + (y-3)^2 = 5^2$.

(b) This circle is more easily given in polar coordinates: $r = 4$. The Cartesian equation is also simple: $x^2 + y^2 = 16$.

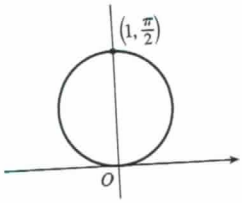
29. $\theta = -\pi/6$



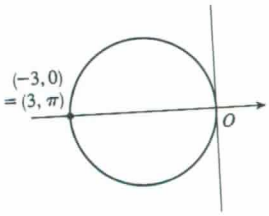
30. $r^2 - 3r + 2 = 0 \iff (r-1)(r-2) = 0 \iff$ $r = 1$ or $r = 2$



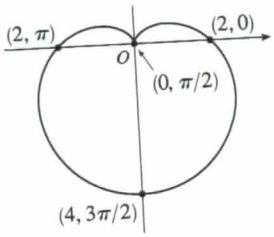
31. $r = \sin\theta \iff r^2 = r\sin\theta \iff x^2 + y^2 = y \iff$ $x^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$. The reasoning here is the same as in Exercise 17. This is a circle of radius $\frac{1}{2}$ centered at $\left(0, \frac{1}{2}\right)$.



32. $r = -3\cos\theta \iff r^2 = -3r\cos\theta \iff$ $x^2 + y^2 = -3x \iff \left(x + \frac{3}{2}\right)^2 + y^2 = \left(\frac{3}{2}\right)^2$. This curve is a circle of radius $\frac{3}{2}$ centered at $\left(-\frac{3}{2}, 0\right)$.



33. $r = 2(1 - \sin\theta)$. This curve is a cardioid.



34. $r = 1 - 3\cos\theta$. This is a limaçon.