Calculus, Early Transcendentals

by James Stewart, 6E.

Appendix D. Trigonometry

pages A24 — A33 (towards back of book)

Solutions

D Trigonometry

1. $210^\circ = 210^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{6}$ rad

2. $300^\circ = 300^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{3}$ rad

3. $9^\circ = 9^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{20}$ rad

4. $-315^\circ = -315^\circ\left(\frac{\pi}{180^\circ}\right) = -\frac{7\pi}{4}$ rad

5. $900^\circ = 900^\circ\left(\frac{\pi}{180^\circ}\right) = 5\pi$ rad

6. $36^\circ = 36^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{5}$ rad

7. 4π rad $= 4\pi\left(\frac{180^\circ}{\pi}\right) = 720^\circ$

8. $-\frac{7\pi}{2}$ rad $= -\frac{7\pi}{2}\left(\frac{180^\circ}{\pi}\right) = -630^\circ$

9. $\frac{5\pi}{12}$ rad $= \frac{5\pi}{12}\left(\frac{180^\circ}{\pi}\right) = 75^\circ$

10. $\frac{8\pi}{3}$ rad $= \frac{8\pi}{3}\left(\frac{180^\circ}{\pi}\right) = 480^\circ$

11. $-\frac{3\pi}{8}$ rad $= -\frac{3\pi}{8}\left(\frac{180^\circ}{\pi}\right) = -67.5^\circ$

12. 5 rad $= 5\left(\frac{180^\circ}{\pi}\right) = \left(\frac{900}{\pi}\right)^\circ$

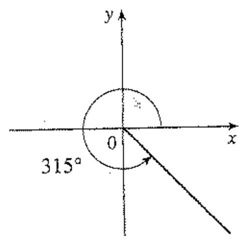
13. Using Formula 3, $a = r\theta = 36 \cdot \frac{\pi}{12} = 3\pi$ cm.

14. Using Formula 3, $a = r\theta = 10 \cdot 72^\circ\left(\frac{\pi}{180^\circ}\right) = 4\pi$ cm.

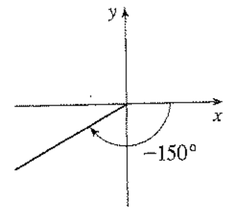
15. Using Formula 3, $\theta = a/r = \frac{1}{1.5} = \frac{2}{3}$ rad $= \frac{2}{3}\left(\frac{180^\circ}{\pi}\right) = \left(\frac{120}{\pi}\right)^\circ \approx 38.2^\circ$.

16. $a = r\theta \quad \Rightarrow \quad r = \frac{a}{\theta} = \frac{6}{3\pi/4} = \frac{8}{\pi}$ cm

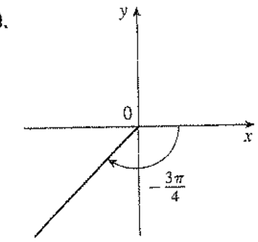
17.



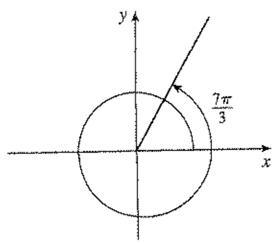
18.



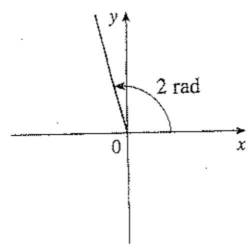
19.



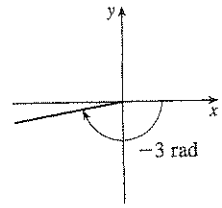
20.



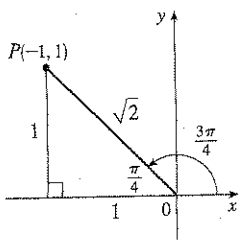
21.



22.

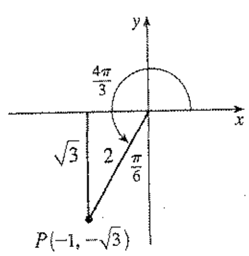


23.



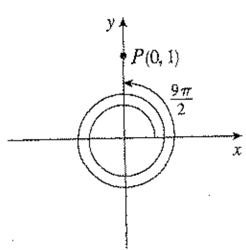
From the diagram we see that a point on the terminal side is $P(-1, 1)$. Therefore, taking $x = -1$, $y = 1$, $r = \sqrt{2}$ in the definitions of the trigonometric ratios, we have $\sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan \frac{3\pi}{4} = -1$, $\csc \frac{3\pi}{4} = \sqrt{2}$, $\sec \frac{3\pi}{4} = -\sqrt{2}$, and $\cot \frac{3\pi}{4} = -1$.

24.



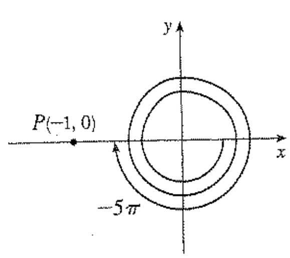
From the diagram and Figure 8, we see that a point on the terminal side is $P(-1, -\sqrt{3})$. Therefore, taking $x = -1$, $y = -\sqrt{3}$, $r = 2$ in the definitions of the trigonometric ratios, we have $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$, $\cos \frac{4\pi}{3} = -\frac{1}{2}$, $\tan \frac{4\pi}{3} = \sqrt{3}$, $\csc \frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$, $\sec \frac{4\pi}{3} = -2$, and $\cot \frac{4\pi}{3} = \frac{1}{\sqrt{3}}$.

25.



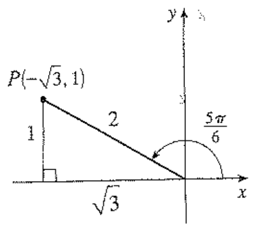
From the diagram we see that a point on the terminal side is $P(0, 1)$. Therefore taking $x = 0$, $y = 1$, $r = 1$ in the definitions of the trigonometric ratios, we have $\sin \frac{9\pi}{2} = 1$, $\cos \frac{9\pi}{2} = 0$, $\tan \frac{9\pi}{2} = y/x$ is undefined since $x = 0$, $\csc \frac{9\pi}{2} = 1$, $\sec \frac{9\pi}{2} = r/x$ is undefined since $x = 0$, and $\cot \frac{9\pi}{2} = 0$.

26.



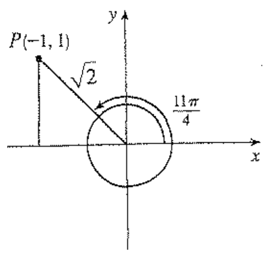
From the diagram, we see that a point on the terminal side is $P(-1, 0)$. Therefore taking $x = -1$, $y = 0$, $r = 1$ in the definitions of the trigonometric ratios we have $\sin(-5\pi) = 0$, $\cos(-5\pi) = -1$, $\tan(-5\pi) = 0$, $\csc(-5\pi)$ is undefined, $\sec(-5\pi) = -1$, and $\cot(-5\pi)$ is undefined.

27.



Using Figure 8 we see that a point on the terminal side is $P(-\sqrt{3}, 1)$. Therefore taking $x = -\sqrt{3}$, $y = 1$, $r = 2$ in the definitions of the trigonometric ratios, we have $\sin \frac{5\pi}{6} = \frac{1}{2}$, $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$, $\tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$, $\csc \frac{5\pi}{6} = 2$, $\sec \frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$, and $\cot \frac{5\pi}{6} = -\sqrt{3}$.

28.



From the diagram, we see that a point on the terminal side is $P(-1, 1)$. Therefore taking $x = -1$, $y = 1$, $r = \sqrt{2}$ in the definitions of the trigonometric ratios we have $\sin \frac{11\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos \frac{11\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan \frac{11\pi}{4} = -1$, $\csc \frac{11\pi}{4} = \sqrt{2}$, $\sec \frac{11\pi}{4} = -\sqrt{2}$, and $\cot \frac{11\pi}{4} = -1$.

29. $\sin \theta = y/r = \frac{3}{5} \Rightarrow y = 3, r = 5$, and $x = \sqrt{r^2 - y^2} = 4$ (since $0 < \theta < \frac{\pi}{2}$). Therefore taking $x = 4$, $y = 3$, $r = 5$ in the definitions of the trigonometric ratios, we have $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = \frac{5}{4}$, and $\cot \theta = \frac{4}{3}$.

30. Since $0 < \alpha < \frac{\pi}{2}$, α is in the first quadrant where x and y are both positive. Therefore, $\tan \alpha = y/x = \frac{2}{1} \Rightarrow y = 2$, $x = 1$, and $r = \sqrt{x^2 + y^2} = \sqrt{5}$. Taking $x = 1$, $y = 2$, $r = \sqrt{5}$ in the definitions of the trigonometric ratios, we have $\sin \alpha = \frac{2}{\sqrt{5}}$, $\cos \alpha = \frac{1}{\sqrt{5}}$, $\csc \alpha = \frac{\sqrt{5}}{2}$, $\sec \alpha = \sqrt{5}$, and $\cot \alpha = \frac{1}{2}$.

31. $\frac{\pi}{2} < \phi < \pi \Rightarrow \phi$ is in the second quadrant, where x is negative and y is positive. Therefore $\sec \phi = r/x = -1.5 = -\frac{3}{2} \Rightarrow r = 3, x = -2$, and $y = \sqrt{r^2 - x^2} = \sqrt{5}$. Taking $x = -2$, $y = \sqrt{5}$, and $r = 3$ in the definitions of the trigonometric ratios, we have $\sin \phi = \frac{\sqrt{5}}{3}$, $\cos \phi = -\frac{2}{3}$, $\tan \phi = -\frac{\sqrt{5}}{2}$, $\csc \phi = \frac{3}{\sqrt{5}}$, and $\cot \theta = -\frac{2}{\sqrt{5}}$.

32. Since $\pi < x < \frac{3\pi}{2}$, x is in the third quadrant where x and y are both negative. Therefore $\cos x = x/r = -\frac{1}{3} \Rightarrow x = -1$, $r = 3$, and $y = -\sqrt{r^2 - x^2} = -\sqrt{8} = -2\sqrt{2}$. Taking $x = -1$, $r = 3$, $y = -2\sqrt{2}$ in the definitions of the trigonometric ratios, we have $\sin x = -\frac{2\sqrt{2}}{3}$, $\tan x = 2\sqrt{2}$, $\csc x = -\frac{3}{2\sqrt{2}}$, $\sec x = -3$, and $\cot x = \frac{1}{2\sqrt{2}}$.

33. $\pi < \beta < 2\pi$ means that β is in the third or fourth quadrant where y is negative. Also since $\cot \beta = x/y = 3$ which is positive, x must also be negative. Therefore $\cot \beta = x/y = \frac{3}{1} \Rightarrow x = -3, y = -1$, and $r = \sqrt{x^2 + y^2} = \sqrt{10}$. Taking $x = -3$, $y = -1$ and $r = \sqrt{10}$ in the definitions of the trigonometric ratios, we have $\sin \beta = -\frac{1}{\sqrt{10}}$, $\cos \beta = -\frac{3}{\sqrt{10}}$, $\tan \beta = \frac{1}{3}$, $\csc \beta = -\sqrt{10}$, and $\sec \beta = -\frac{\sqrt{10}}{3}$.

34. Since $\frac{3\pi}{2} < \theta < 2\pi$, θ is in the fourth quadrant where x is positive and y is negative. Therefore $\csc \theta = r/y = -\frac{4}{3} \Rightarrow r = 4, y = -3$, and $x = \sqrt{r^2 - y^2} = \sqrt{7}$. Taking $x = \sqrt{7}$, $y = -3$, and $r = 4$ in the definitions of the trigonometric ratios, we have $\sin \theta = -\frac{3}{4}$, $\cos \theta = \frac{\sqrt{7}}{4}$, $\tan \theta = -\frac{3}{\sqrt{7}}$, $\sec \theta = \frac{4}{\sqrt{7}}$, and $\cot \theta = -\frac{\sqrt{7}}{3}$.

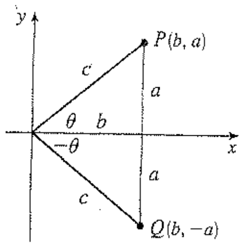
35. $\sin 35^\circ = \dfrac{x}{10} \quad \Rightarrow \quad x = 10 \sin 35^\circ \approx 5.73576 \text{ cm}$

36. $\cos 40^\circ = \dfrac{x}{25} \quad \Rightarrow \quad x = 25 \cos 40^\circ \approx 19.15111 \text{ cm}$

37. $\tan \frac{2\pi}{5} = \dfrac{x}{8} \quad \Rightarrow \quad x = 8 \tan \frac{2\pi}{5} \approx 24.62147 \text{ cm}$

38. $\cos \dfrac{3\pi}{8} = \dfrac{22}{x} \quad \Rightarrow \quad x = \dfrac{22}{\cos \frac{3\pi}{8}} \approx 57.48877 \text{ cm}$

39.



(a) From the diagram we see that $\sin\theta = \dfrac{y}{r} = \dfrac{a}{c}$, and $\sin(-\theta) = \dfrac{-a}{c} = -\dfrac{a}{c} = -\sin\theta$.

(b) Again from the diagram we see that $\cos\theta = \dfrac{x}{r} = \dfrac{b}{c} = \cos(-\theta)$.

40. (a) Using (12a) and (12b), we have

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x\,\cos y + \cos x\,\sin y}{\cos x\,\cos y - \sin x\,\sin y} = \frac{\dfrac{\sin x\,\cos y}{\cos x\,\cos y} + \dfrac{\cos x\,\sin y}{\cos x\,\cos y}}{\dfrac{\cos x\,\cos y}{\cos x\,\cos y} - \dfrac{\sin x\,\sin y}{\cos x\,\cos y}} = \frac{\tan x + \tan y}{1 - \tan x\,\tan y}$$

(b) From (10a) and (10b), we have $\tan(-\theta) = -\tan\theta$, so (14a) implies that

$$\tan(x-y) = \tan(x+(-y)) = \frac{\tan x + \tan(-y)}{1 - \tan x\,\tan(-y)} = \frac{\tan x - \tan y}{1 + \tan x\,\tan y}$$

41. (a) Using (12a) and (13a), we have

$$\tfrac{1}{2}[\sin(x+y) + \sin(x-y)] = \tfrac{1}{2}[\sin x\,\cos y + \cos x\,\sin y + \sin x\,\cos y - \cos x\,\sin y] = \tfrac{1}{2}(2\sin x\,\cos y) = \sin x\,\cos y.$$

(b) This time, using (12b) and (13b), we have

$$\tfrac{1}{2}[\cos(x+y) + \cos(x-y)] = \tfrac{1}{2}[\cos x\,\cos y - \sin x\,\sin y + \cos x\,\cos y + \sin x\,\sin y] = \tfrac{1}{2}(2\cos x\,\cos y) = \cos x\,\cos y.$$

(c) Again using (12b) and (13b), we have

$$\begin{aligned}\tfrac{1}{2}[\cos(x-y) - \cos(x+y)] &= \tfrac{1}{2}[\cos x\,\cos y + \sin x\,\sin y - \cos x\,\cos y + \sin x\,\sin y] \\ &= \tfrac{1}{2}(2\sin x\,\sin y) = \sin x\,\sin y\end{aligned}$$

42. Using (13b), $\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x = 0\cdot\cos x + 1\cdot\sin x = \sin x$.

43. Using (12a), we have $\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x = 1\cdot\cos x + 0\cdot\sin x = \cos x$.

44. Using (13a), we have $\sin(\pi - x) = \sin\pi\cos x - \cos\pi\,\sin x = 0\cdot\cos x - (-1)\sin x = \sin x$.

45. Using (6), we have $\sin\theta\cot\theta = \sin\theta\cdot\dfrac{\cos\theta}{\sin\theta} = \cos\theta$.

46. $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\,\cos x + \cos^2 x = (\sin^2 x + \cos^2 x) + \sin 2x$ [by (15a)] $= 1 + \sin 2x$ [by (7)]

47. $\sec y - \cos y = \dfrac{1}{\cos y} - \cos y$ [by (6)] $= \dfrac{1-\cos^2 y}{\cos y} = \dfrac{\sin^2 y}{\cos y}$ [by (7)] $= \dfrac{\sin y}{\cos y}\sin y = \tan y\,\sin y$ [by (6)]

48. $\tan^2\alpha - \sin^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha} - \sin^2\alpha = \dfrac{\sin^2\alpha - \sin^2\alpha\,\cos^2\alpha}{\cos^2\alpha} = \dfrac{\sin^2\alpha\,(1-\cos^2\alpha)}{\cos^2\alpha} = \tan^2\alpha\,\sin^2\alpha$ [by (6), (7)]

49. $\cot^2\theta + \sec^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}$ [by (6)] $= \dfrac{\cos^2\theta\,\cos^2\theta + \sin^2\theta}{\sin^2\theta\,\cos^2\theta}$

$= \dfrac{(1-\sin^2\theta)(1-\sin^2\theta)+\sin^2\theta}{\sin^2\theta\cos^2\theta}$ [by (7)] $= \dfrac{1-\sin^2\theta+\sin^4\theta}{\sin^2\theta\cos^2\theta}$

$= \dfrac{\cos^2\theta+\sin^4\theta}{\sin^2\theta\cos^2\theta}$ [*by* (7)] $= \dfrac{1}{\sin^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \csc^2\theta + \tan^2\theta$ [by (6)]

50. $2\csc 2t = \dfrac{2}{\sin 2t} = \dfrac{2}{2\sin t\,\cos t}$ [by (15a)] $= \dfrac{1}{\sin t\,\cos t} = \sec t\,\csc t$

51. Using (14a), we have $\tan 2\theta = \tan(\theta+\theta) = \dfrac{\tan\theta+\tan\theta}{1-\tan\theta\,\tan\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

52. $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = \dfrac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} = \dfrac{2}{1-\sin^2\theta} = \dfrac{2}{\cos^2\theta}$ [by (7)] $= 2\sec^2\theta$

53. Using (15a) and (16a),

$$\sin x\,\sin 2x + \cos x\,\cos 2x = \sin x\,(2\sin x\cos x) + \cos x\,(2\cos^2 x - 1) = 2\sin^2 x\,\cos x + 2\cos^3 x - \cos x$$

$= 2(1-\cos^2 x)\,\cos x + 2\cos^3 x - \cos x$ [by (7)]

$= 2\cos x - 2\cos^3 x + 2\cos^3 x - \cos x = \cos x$

Or: $\sin x\sin 2x + \cos x\,\cos 2x = \cos(2x - x)$ [by 13(b)] $= \cos x$

54. We start with the right side using equations (12a) and (13a):

$$\sin(x+y)\sin(x-y) = (\sin x\,\cos y + \cos x\,\sin y)(\sin x\,\cos y - \cos x\,\sin y)$$

$= \sin^2 x\,\cos^2 y - \sin x\,\cos y\,\cos x\,\sin y + \cos x\,\sin y\,\sin x\,\cos y - \cos^2 x\,\sin^2 y$

$= \sin^2 x\,(1-\sin^2 y) - (1-\sin^2 x)\,\sin^2 y$ [by (7)]

$= \sin^2 x - \sin^2 x\,\sin^2 y - \sin^2 y + \sin^2 x\,\sin^2 y = \sin^2 x - \sin^2 y$

55. $\dfrac{\sin\phi}{1-\cos\phi} = \dfrac{\sin\phi}{1-\cos\phi}\cdot\dfrac{1+\cos\phi}{1+\cos\phi} = \dfrac{\sin\phi\,(1+\cos\phi)}{1-\cos^2\phi} = \dfrac{\sin\phi\,(1+\cos\phi)}{\sin^2\phi}$ [by (7)]

$= \dfrac{1+\cos\phi}{\sin\phi} = \dfrac{1}{\sin\phi} + \dfrac{\cos\phi}{\sin\phi} = \csc\phi + \cot\phi$ [by (6)]

56. $\tan x + \tan y = \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y} = \dfrac{\sin x\,\cos y + \cos x\,\sin y}{\cos x\,\cos y} = \dfrac{\sin(x+y)}{\cos x\,\cos y}$ [by (12a)]

57. Using (12a),

$$\sin 3\theta + \sin\theta = \sin(2\theta+\theta) + \sin\theta = \sin 2\theta\,\cos\theta + \cos 2\theta\,\sin\theta + \sin\theta$$

$= \sin 2\theta\cos\theta + (2\cos^2\theta - 1)\,\sin\theta + \sin\theta$ [by (16a)]

$= \sin 2\theta\,\cos\theta + 2\cos^2\theta\,\sin\theta - \sin\theta + \sin\theta = \sin 2\theta\,\cos\theta + \sin 2\theta\,\cos\theta$ [by (15a)]

$= 2\sin 2\theta\,\cos\theta$

58. We use (12b) with $x = 2\theta$, $y = \theta$ to get

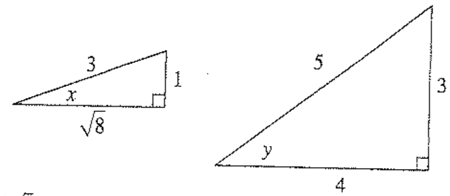
$$\cos 3\theta = \cos(2\theta+\theta) = \cos 2\theta\,\cos\theta - \sin 2\theta\,\sin\theta$$

$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\,\cos\theta$ [by (16a) and (15a)]

$= (2\cos^2\theta - 1)\cos\theta - 2(1-\cos^2\theta)\,\cos\theta$ [by (7)]

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta = 4\cos^3\theta - 3\cos\theta$

59. Since $\sin x = \frac{1}{3}$ we can label the opposite side as having length 1, the hypotenuse as having length 3, and use the Pythagorean Theorem to get that the adjacent side has length $\sqrt{8}$. Then, from the diagram, $\cos x = \frac{\sqrt{8}}{3}$. Similarly we have that $\sin y = \frac{3}{5}$. Now use (12a):



$\sin(x+y) = \sin x \cos y + \cos x \sin y = \frac{1}{3} \cdot \frac{4}{5} + \frac{\sqrt{8}}{3} \cdot \frac{3}{5} = \frac{4}{15} + \frac{3\sqrt{8}}{15} = \frac{4+6\sqrt{2}}{15}$.

60. Use (12b) and the values for $\sin y$ and $\cos x$ obtained in Exercise 59 to get

$$\cos(x+y) = \cos x \cos y - \sin x \sin y = \frac{\sqrt{8}}{3} \cdot \frac{4}{5} - \frac{1}{3} \cdot \frac{3}{5} = \frac{8\sqrt{2}-3}{15}$$

61. Using (13b) and the values for $\cos x$ and $\sin y$ obtained in Exercise 59, we have

$$\cos(x-y) = \cos x \cos y + \sin x \sin y = \frac{\sqrt{8}}{3} \cdot \frac{4}{5} + \frac{1}{3} \cdot \frac{3}{5} = \frac{8\sqrt{2}+3}{15}$$

62. Using (13a) and the values for $\sin y$ and $\cos x$ obtained in Exercise 59, we get

$$\sin(x-y) = \sin x \cos y - \cos x \sin y = \frac{1}{3} \cdot \frac{4}{5} - \frac{\sqrt{8}}{3} \cdot \frac{3}{5} = \frac{4-6\sqrt{2}}{15}$$

63. Using (15a) and the values for $\sin y$ and $\cos y$ obtained in Exercise 59, we have $\sin 2y = 2\sin y \cos y = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$.

64. Using (16a) with $\cos y = \frac{4}{5}$, we have $\cos 2y = 2\cos^2 y - 1 = 2\left(\frac{4}{5}\right)^2 - 1 = \frac{32}{25} - 1 = \frac{7}{25}$.

65. $2\cos x - 1 = 0 \Leftrightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$ for $x \in [0, 2\pi]$.

66. $3\cot^2 x = 1 \Leftrightarrow 3 = 1/\cot^2 x \Leftrightarrow \tan^2 x = 3 \Leftrightarrow \tan x = \pm\sqrt{3} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

67. $2\sin^2 x = 1 \Leftrightarrow \sin^2 x = \frac{1}{2} \Leftrightarrow \sin x = \pm\frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

68. $|\tan x| = 1 \Leftrightarrow \tan x = -1$ or $\tan x = 1 \Leftrightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$ or $x = \frac{\pi}{4}, \frac{5\pi}{4}$.

69. Using (15a), we have $\sin 2x = \cos x \Leftrightarrow 2\sin x \cos x - \cos x = 0 \Leftrightarrow \cos x(2\sin x - 1) = 0 \Leftrightarrow \cos x = 0$ or $2\sin x - 1 = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. Therefore, the solutions are $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$.

70. By (15a), $2\cos x + \sin 2x = 0 \Leftrightarrow 2\cos x + 2\sin x \cos x = 0 \Leftrightarrow 2\cos x(1 + \sin x) = 0 \Leftrightarrow \cos x = 0$ or $1 + \sin x = 0 \Leftrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\sin x = -1 \Rightarrow x = \frac{3}{2}\pi$. So the solutions are $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

71. $\sin x = \tan x \Leftrightarrow \sin x - \tan x = 0 \Leftrightarrow \sin x - \dfrac{\sin x}{\cos x} = 0 \Leftrightarrow \sin x\left(1 - \dfrac{1}{\cos x}\right) = 0 \Leftrightarrow \sin x = 0$ or $1 - \dfrac{1}{\cos x} = 0 \Rightarrow x = 0, \pi, 2\pi$ or $1 = \dfrac{1}{\cos x} \Rightarrow \cos x = 1 \Rightarrow x = 0, 2\pi$. Therefore the solutions are $x = 0, \pi, 2\pi$.

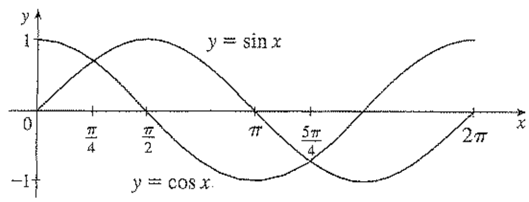
72. By (16a), $2 + \cos 2x = 3\cos x \Leftrightarrow 2 + 2\cos^2 x - 1 = 3\cos x \Leftrightarrow 2\cos^2 x - 3\cos x + 1 = 0 \Leftrightarrow (2\cos x - 1)(\cos x - 1) = 0 \Leftrightarrow \cos x = 1$ or $\cos x = \frac{1}{2} \Rightarrow x = 0, 2\pi$ or $x = \frac{\pi}{3}, \frac{5\pi}{3}$.

73. We know that $\sin x = \frac{1}{2}$ when $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, and from Figure 13(a), we see that $\sin x \le \frac{1}{2} \Rightarrow 0 \le x \le \frac{\pi}{6}$ or $\frac{5\pi}{6} \le x \le 2\pi$ for $x \in [0, 2\pi]$.

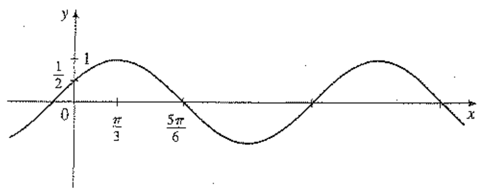
74. $2\cos x + 1 > 0 \Rightarrow 2\cos x > -1 \Rightarrow \cos x > -\frac{1}{2}$. $\cos x = -\frac{1}{2}$ when $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ and from Figure 13(b), we see that $\cos x > -\frac{1}{2}$ when $0 \le x < \frac{2\pi}{3}, \frac{4\pi}{3} < x \le 2\pi$.

75. $\tan x = -1$ when $x = \frac{3\pi}{4}, \frac{7\pi}{4}$, and $\tan x = 1$ when $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$. From Figure 14 we see that $-1 < \tan x < 1 \Rightarrow$ $0 \le x < \frac{\pi}{4}$, $\frac{3\pi}{4} < x < \frac{5\pi}{4}$, and $\frac{7\pi}{4} < x \le 2\pi$.

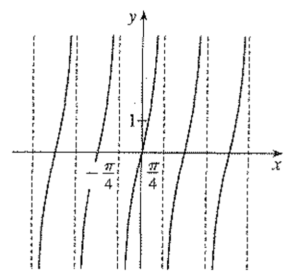
76. We know that $\sin x = \cos x$ when $x = \frac{\pi}{4}, \frac{5\pi}{4}$, and from the diagram we see that $\sin x > \cos x$ when $\frac{\pi}{4} < x < \frac{5\pi}{4}$.



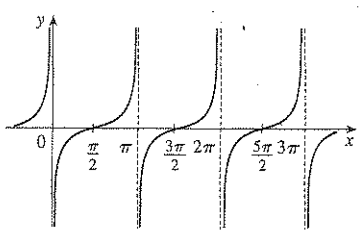
77. $y = \cos\left(x - \frac{\pi}{3}\right)$. We start with the graph of $y = \cos x$ and shift it $\frac{\pi}{3}$ units to the right.



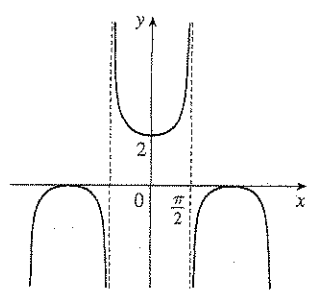
78. $y = \tan 2x$. Start with the graph of $y = \tan x$ with period π and compress it to a period of $\frac{\pi}{2}$.



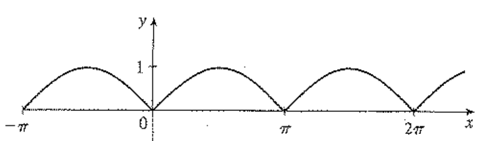
79. $y = \frac{1}{3}\tan\left(x - \frac{\pi}{2}\right)$. We start with the graph of $y = \tan x$, shift it $\frac{\pi}{2}$ units to the right and compress it to $\frac{1}{3}$ of its original vertical size.



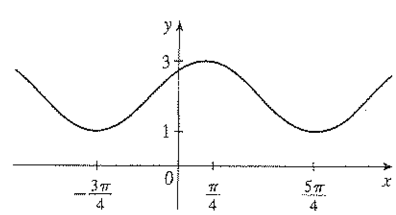
80. $y = 1 + \sec x$. Start with the graph of $y = \sec x$ and raise it by one unit.



81. $y = |\sin x|$. We start with the graph of $y = \sin x$ and reflect the parts below the x-axis about the x-axis.



82. $y = 2 + \sin\left(x + \frac{\pi}{4}\right)$. Start with the graph of $y = \sin x$, and shift it $\frac{\pi}{4}$ units to the left and 2 units up.



83. From the figure in the text, we see that $x = b\cos\theta$, $y = b\sin\theta$, and from the distance formula we have that the distance c from (x, y) to $(a, 0)$ is $c = \sqrt{(x-a)^2 + (y-0)^2} \Rightarrow$

$$c^2 = (b\cos\theta - a)^2 + (b\sin\theta)^2 = b^2\cos^2\theta - 2ab\cos\theta + a^2 + b^2\sin^2\theta$$
$$= a^2 + b^2(\cos^2\theta + \sin^2\theta) - 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta \quad \text{[by (7)]}$$

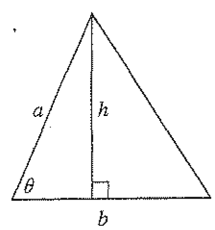
84. $|AB|^2 = |AC|^2 + |BC|^2 - 2\,|AC|\,|BC| \cos \angle C = (820)^2 + (910)^2 - 2(820)(910) \cos 103° \approx 1{,}836{,}217 \Rightarrow$
$|AB| \approx 1355$ m

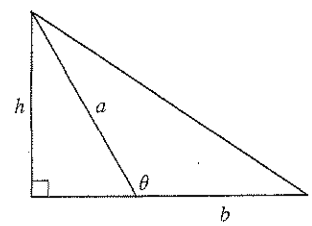
85. Using the Law of Cosines, we have $c^2 = 1^2 + 1^2 - 2(1)(1)\cos(\alpha - \beta) = 2\,[1 - \cos(\alpha - \beta)]$. Now, using the distance formula, $c^2 = |AB|^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$. Equating these two expressions for c^2, we get
$2[1 - \cos(\alpha - \beta)] = \cos^2\alpha + \sin^2\alpha + \cos^2\beta + \sin^2\beta - 2\cos\alpha\,\cos\beta - 2\sin\alpha\,\sin\beta \Rightarrow$
$1 - \cos(\alpha - \beta) = 1 - \cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta \Rightarrow \cos(\alpha - \beta) = \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta$.

86. $\cos(x + y) = \cos(x - (-y)) = \cos x\,\cos(-y) + \sin x\,\sin(-y)$
$= \cos x\,\cos y - \sin x\,\sin y$ [using Equations (10a) and (10b)]

87. In Exercise 86 we used the subtraction formula for cosine to prove the addition formula for cosine. Using that formula with $x = \frac{\pi}{2} - \alpha$, $y = \beta$, we get $\cos\left[\left(\frac{\pi}{2} - \alpha\right) + \beta\right] = \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta \Rightarrow$
$\cos\left[\frac{\pi}{2} - (\alpha - \beta)\right] = \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta$. Now we use the identities given in the problem, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ and $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$, to get $\sin(\alpha - \beta) = \sin\alpha\,\cos\beta - \cos\alpha\,\sin\beta$.

88. If $0 < \theta < \frac{\pi}{2}$, we have the case depicted in the first diagram. In this case, we see that the height of the triangle is $h = a\sin\theta$. If $\frac{\pi}{2} \le \theta < \pi$, we have the case depicted in the second diagram. In this case, the height of the triangle is $h = a\sin(\pi - \theta) = a\sin\theta$ (by the identity proved in Exercise 44). So in either case, the area of the triangle is $\frac{1}{2}bh = \frac{1}{2}ab\sin\theta$.





89. Using the formula from part (a), the area of the triangle is $\frac{1}{2}(10)(3)\sin 107° \approx 14.34457\ \text{cm}^2$.