Solution 10.8 # 63 Anton et al 8^{th} ed.

63. Prove: If the interval of convergence of the series $\sum_{k=0}^{\infty} c_k(x-x_0)^k$ is $(x_0 - R, x_0 + R]$, then the series converges conditionally at $x_0 + R$.

63. The assumption is that $\sum_{k=0}^{\infty} c_k R^k$ is convergent and $\sum_{k=0}^{\infty} c_k(-R)^k$ is divergent. Suppose that $\sum_{k=0}^{\infty} c_k R^k$ is absolutely convergent then $\sum_{k=0}^{\infty} c_k(-R)^k$ is also absolutely convergent and hence convergent because $|c_k R^k| = |c_k(-R)^k|$, which contradicts the assumption that $\sum_{k=0}^{\infty} c_k(-R)^k$ is divergent so $\sum_{k=0}^{\infty} c_k R^k$ must be conditionally convergent.