

Integration by Substitution - a review

1. $\int (4 - 2x)^3 dx$

2. $\int 3\sqrt{4 + 2x} dx$

3. $\int x \sec^2(x^2) dx$

4. $\int 4x \tan(x^2) dx$

5. $\int \frac{\sin 3x}{2 + \cos 3x} dx$

6. $\int \frac{1}{9 + 4x^2} dx$

~~7. $\int e^x \sinh(e^x) dx$~~

8. $\int \frac{\sec(\ln x) \tan(\ln x)}{x} dx$

9. $\int e^{\tan x} \sec^2 x dx$

10. $\int \frac{x}{\sqrt{1 - x^4}} dx$

11. $\int \cos^5 5x \sin 5x dx$

~~12. $\int \frac{\cos x}{\sin x \sqrt{\sin^2 x + 1}} dx$~~

~~13. $\int \frac{e^x}{\sqrt{4 + e^{2x}}} dx$~~

14. $\int \frac{e^{\tan^{-1} x}}{1 + x^2} dx$

15. $\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}} dx$

16. $\int (x + 1) \cot(x^2 + 2x) dx$

~~17. $\int \frac{\cosh \sqrt{x}}{\sqrt{x}} dx$~~

18. $\int \frac{dx}{x(\ln x)^2}$

19. $\int \frac{dx}{\sqrt{x} 3^{\sqrt{x}}}$

20. $\int \sec(\sin \theta) \tan(\sin \theta) \cos \theta d\theta$

~~21. $\int \frac{\operatorname{csch}^2(2/x)}{x^2} dx$~~

22. $\int \frac{dx}{\sqrt{x^2 - 4}}$

~~23. $\int \frac{e}{4 - e^{-2x}} dx$~~

24. $\int \frac{\cos(\ln x)}{x} dx$

25. $\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx$

~~26. $\int \frac{\sinh(x^{-1/2})}{x^{3/2}} dx$~~

27. $\int \frac{x}{\csc(x^2)} dx$

28. $\int \frac{e^x}{\sqrt{4 - e^{2x}}} dx$

29. $\int x 4^{-x^2} dx$

30. $\int 2^{\pi x} dx$

2 → (obviously) omit problems crossed out.

2 → From § 8.1 of Anton et al. book, 8th ed.

A review of

Integration by Substitution

Sol'n

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Solutions ☺

from § 8.1 of Anton et. al. book, 8th edition

1. $u = 4 - 2x, du = -2dx, -\frac{1}{2} \int u^3 du = -\frac{1}{8}u^4 + C = -\frac{1}{8}(4 - 2x)^4 + C$
2. $u = 4 + 2x, du = 2dx, \frac{3}{2} \int \sqrt{u} du = u^{3/2} + C = (4 + 2x)^{3/2} + C$
3. $u = x^2, du = 2xdx, \frac{1}{2} \int \sec^2 u du = \frac{1}{2} \tan u + C = \frac{1}{2} \tan(x^2) + C$
4. $u = x^2, du = 2xdx, 2 \int \tan u du = -2 \ln |\cos u| + C = -2 \ln |\cos(x^2)| + C$
5. $u = 2 + \cos 3x, du = -3 \sin 3x dx, -\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ln |u| + C = -\frac{1}{3} \ln(2 + \cos 3x) + C$
6. $u = \frac{2}{3}x, du = \frac{2}{3}dx, \frac{1}{6} \int \frac{du}{1+u^2} = \frac{1}{6} \tan^{-1} u + C = \frac{1}{6} \tan^{-1} \frac{2}{3}x + C$
- ~~7.~~ $u = e^x, du = e^x dx, \int \sinh u du = \cosh u + C = \cosh e^x + C$
8. $u = \ln x, du = \frac{1}{x} dx, \int \sec u \tan u du = \sec u + C = \sec(\ln x) + C$
9. $u = \tan x, du = \sec^2 x dx, \int e^u du = e^u + C = e^{\tan x} + C$
10. $u = x^2, du = 2xdx, \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \sin^{-1} u + C = \frac{1}{2} \sin^{-1}(x^2) + C$
11. $u = \cos 5x, du = -5 \sin 5x dx, -\frac{1}{5} \int u^5 du = -\frac{1}{30}u^6 + C = -\frac{1}{30} \cos^6 5x + C$
- ~~12.~~ $u = \sin x, du = \cos x dx, \int \frac{du}{u\sqrt{u^2+1}} = -\ln \left| \frac{1+\sqrt{1+u^2}}{u} \right| + C = -\ln \left| \frac{1+\sqrt{1+\sin^2 x}}{\sin x} \right| + C$
- ~~13.~~ $u = e^x, du = e^x dx, \int \frac{du}{\sqrt{4+u^2}} = \ln(u + \sqrt{u^2+4}) + C = \ln(e^x + \sqrt{e^{2x}+4}) + C$
14. $u = \tan^{-1} x, du = \frac{1}{1+x^2} dx, \int e^u du = e^u + C = e^{\tan^{-1} x} + C$
15. $u = \sqrt{x-1}, du = \frac{1}{2\sqrt{x-1}} dx, 2 \int e^u du = 2e^u + C = 2e^{\sqrt{x-1}} + C$
16. $u = x^2 + 2x, du = (2x+2)dx, \frac{1}{2} \int \cot u du = \frac{1}{2} \ln |\sin u| + C = \frac{1}{2} \ln \sin |x^2 + 2x| + C$
- ~~17.~~ $u = \sqrt{x}, du = \frac{1}{2\sqrt{x}} dx, \int 2 \cosh u du = 2 \sinh u + C = 2 \sinh \sqrt{x} + C$

18. $u = \ln x, du = \frac{dx}{x}, \int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$
19. $u = \sqrt{x}, du = \frac{1}{2\sqrt{x}} dx, \int \frac{2 du}{3^u} = 2 \int e^{-u \ln 3} du = -\frac{2}{\ln 3} e^{-u \ln 3} + C = -\frac{2}{\ln 3} 3^{-\sqrt{x}} + C$
20. $u = \sin \theta, du = \cos \theta d\theta, \int \sec u \tan u du = \sec u + C = \sec(\sin \theta) + C$
- ~~21.~~ $u = \frac{2}{x}, du = -\frac{2}{x^2} dx, -\frac{1}{2} \int \operatorname{csch}^2 u du = \frac{1}{2} \coth u + C = \frac{1}{2} \coth \frac{2}{x} + C$
22. $\int \frac{dx}{\sqrt{x^2 - 4}} = \ln |x + \sqrt{x^2 - 4}| + C$
- ~~23.~~ $u = e^{-x}, du = -e^{-x} dx, -\int \frac{du}{4 - u^2} = -\frac{1}{4} \ln \left| \frac{2 + u}{2 - u} \right| + C = -\frac{1}{4} \ln \left| \frac{2 + e^{-x}}{2 - e^{-x}} \right| + C$
24. $u = \ln x, du = \frac{1}{x} dx, \int \cos u du = \sin u + C = \sin(\ln x) + C$
25. $u = e^x, du = e^x dx, \int \frac{e^x dx}{\sqrt{1 - e^{2x}}} = \int \frac{du}{\sqrt{1 - u^2}} = \sin^{-1} u + C = \sin^{-1} e^x + C$
- ~~26.~~ $u = x^{-1/2}, du = -\frac{1}{2x^{3/2}} dx, -\int 2 \sinh u du = -2 \cosh u + C = -2 \cosh(x^{-1/2}) + C$
27. $u = x^2, du = 2x dx, \frac{1}{2} \int \frac{du}{\csc u} = \frac{1}{2} \int \sin u du = -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos(x^2) + C$
28. $2u = e^x, 2du = e^x dx, \int \frac{2du}{\sqrt{4 - 4u^2}} = \sin^{-1} u + C = \sin^{-1}(e^x/2) + C$
29. $4^{-x^2} = e^{-x^2 \ln 4}, u = -x^2 \ln 4, du = -2x \ln 4 dx = -x \ln 16 dx,$
 $-\frac{1}{\ln 16} \int e^u du = -\frac{1}{\ln 16} e^u + C = -\frac{1}{\ln 16} e^{-x^2 \ln 4} + C = -\frac{1}{\ln 16} 4^{-x^2} + C$
30. $2^{\pi x} = e^{\pi x \ln 2}, \int 2^{\pi x} dx = \frac{1}{\pi \ln 2} e^{\pi x \ln 2} + C = \frac{1}{\pi \ln 2} 2^{\pi x} + C$