§ 11.5 Wording for problems # 23 - 30.

① Is the series an alternating series $\Sigma (-1)^n u_n$ with $u_n > 0$?
If yes, identify u_n.

② If yes for ① ... does the series satisfy the conditions of the AST?
Verify your answer.

③ If yes for ② ... the. Alternating Series Estimation Theorem (ASET) guarantees that the sum of the first N = ___ terms of the series will approximate the infinite series within the given error. (Find the best N).

<u>Remark</u>. If the problem says "correct to "k" decimal places",
estimate w/in $|\text{error}| < 5 \cdot 10^{-(k+1)}$.

<u>Example from class</u>

$$\sum_{n=0}^{\infty} (-1)^n \frac{4}{2n+1} \left[\frac{1}{2^{2n+1}} + \frac{1}{3^{2n+1}} \right]$$

accurate to 3 decimal places
⇓
$|\text{error}| < 5 \cdot 10^{-4} = 0.0005$

① Yes. $u_n = \frac{4}{2n+1} \left[\frac{1}{2^{2n+1}} + \frac{1}{3^{2n+1}} \right] > 0$

② Yes.

(i) $\lim\limits_{n\to\infty} u_n = 0$ ✓ yes clear "$\cancel{\frac{1}{\infty} = 0}$"

(ii) u_n dec? i.e. $u_n > u_{n+1}$. ✓ yes clear.

recall ... was given $|\text{error}| < 0.\underbrace{000}_{3}5$

3 Chart to help → these 2 columns not needed but helpful.

n	$(-1)^n$	$2n+1$	u_n
0	1	1	$4\left(\frac{1}{2}+\frac{1}{3}\right) = 3.\overline{3}$
1	-1	3	$\frac{4}{3}\left(\frac{1}{2^3}+\frac{1}{3^3}\right) \simeq 0.2160493827$
2	1	5	
3	-1	7	
4	1	9	$\frac{4}{9}\left(\frac{1}{2^9}+\frac{1}{3^9}\right) \simeq 8.906356726E^{-4} \simeq 0.00089 \not< 0.0005$
5	-1	11	$\frac{4}{11}\left(\frac{1}{2^{11}}+\frac{1}{3^{11}}\right) \approx 1.796695561E^{-4} \simeq 0.0001796 < 0.0005$

So, by ASET,

$$\left| \sum_{n=0}^{\infty} (-1)^n \frac{4}{2n+1}\left(\frac{1}{2^{n+1}}+\frac{1}{3^{n+1}}\right) - \sum_{n=0}^{4} (-1)^n \frac{4}{2n+1}\left(\frac{1}{2^{n+1}}+\frac{1}{3^{n+1}}\right) \right|$$

$$< \frac{4}{11}\left(\frac{1}{2^{11}}+\frac{1}{3^{11}}\right) < 0.0005 .$$

Note

$$\sum_{n=0}^{4} (-1)^n u_n = \underbrace{u_0 - u_1 + u_2 - u_3 + u_4}_{5 \text{ terms.}}$$

Answer $N = 5$.