

Our Math 142 Course Homepage (CH) is <http://people.math.sc.edu/girardi/w142.html>

Generalized Exponential  $y = b^x$  and Logarithmic  $y = \log_b x$  Functions  
with base  $b$  where  $0 < b \neq 1$ . Also  $\ln \equiv \log_e$ .

$$f(x) = b^x \equiv e^{x \ln b} : (-\infty, \infty) \rightarrow (0, \infty)$$

$$g(x) = \log_b x \equiv \text{the inverse of the fn. } f(x) = b^x : (0, \infty) \rightarrow (-\infty, \infty)$$

$$y = \log_b x \iff x = b^y$$

$$(\log_a b)(\log_b c) = \log_a c \implies \log_a x = \frac{\ln x}{\ln a}$$

$$x, y > 0 \text{ & } r \in \mathbb{R}$$

$$b^{\log_b x} = x$$

$$\log_b 1 = 0$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$$

$$\log_b(x^r) = r(\log_b x)$$

$$x, y \in \mathbb{R} \text{ & } r \in \mathbb{R} \text{ & } a > 0$$

$$\log_b(b^x) = x$$

$$b^0 = 1$$

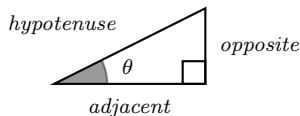
$$b^x b^y = b^{x+y}$$

$$\frac{b^x}{b^y} = b^{x-y}$$

$$(b^x)^r = b^{xr} \quad [\text{recall order of operation: } b^{x^r} = b^{(x^r)}]$$

$$(ab)^x = a^x b^x \quad \text{and} \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

### Basic Trig



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Basic Inverse Trig Functions

$y = \sin \theta$	$\Leftrightarrow$	$\theta = \sin^{-1} y$	where	$-1 \leq y \leq 1$	and	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
$y = \cos \theta$	$\Leftrightarrow$	$\theta = \cos^{-1} y$	where	$-1 \leq y \leq 1$	and	$0 \leq \theta \leq \pi$
$y = \tan \theta$	$\Leftrightarrow$	$\theta = \tan^{-1} y$	where	$y \in \mathbb{R}$	and	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$y = \cot \theta$	$\Leftrightarrow$	$\theta = \cot^{-1} y$	where	$y \in \mathbb{R}$	and	$0 < \theta < \pi$
$y = \sec \theta$	$\Leftrightarrow$	$\theta = \sec^{-1} y$	where	$ y  \geq 1$	and	$0 \leq \theta \leq \pi, \theta \neq \frac{\pi}{2}$
$y = \csc \theta$	$\Leftrightarrow$	$\theta = \csc^{-1} y$	where	$ y  \geq 1$	and	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \theta \neq 0$

Fundamental Theorem of Calculus (FTC)

Let  $f: [a, b] \rightarrow \mathbb{R}$  be a continuous function.

Let  $F: [a, b] \rightarrow \mathbb{R}$  be a function.

- If  $F$  is an antiderivative of  $f$  on  $[a, b]$  (i.e.  $F'(x) = f(x)$  for each  $x \in [a, b]$ ), then

$$\int_a^b f(x) dx \equiv \int_a^b F'(x) dx = F(b) - F(a) .$$

- If  $F(x) = \int_a^x f(t) dt$  for each  $x \in [a, b]$ , then  $F$  is an antiderivative of  $f$  on  $[a, b]$ , i.e.

$$F'(x) \equiv D_x \left[ \int_a^x f(t) dt \right] = f(x) .$$

Basic Differentiation Rules

Let  $y = f(x)$  and  $y = g(x)$  be functions which are differentiable at  $x$ . Let  $a$  and  $b$  are constants.

$$\begin{aligned} D_x [af(x) + bg(x)] &= af'(x) + bg'(x) \\ D_x [f(x) \cdot g(x)] &= f'(x) \cdot g(x) + f(x) \cdot g'(x) \end{aligned}$$

In the that case  $g(x) \neq 0$

$$D_x \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2} .$$

In the case that  $f$  is differentiable at  $x$  and  $g$  is differentiable at  $f(x)$

$$D_x [g(f(x))] = g'(f(x)) f'(x) .$$

Hyperbolic Trig Functions<sup>1</sup>

$$\begin{array}{lll} \cosh x = \frac{e^x + e^{-x}}{2} & \sinh x = \frac{e^x - e^{-x}}{2} & \cosh^2 x - \sinh^2 x = 1 \\ \tanh x = \frac{\sinh x}{\cosh x} & \coth x = \frac{\cosh x}{\sinh x} & \operatorname{sech} x = \frac{1}{\cosh x} \quad \operatorname{csch} x = \frac{1}{\sinh x} \end{array}$$

DERIVATIVES

$$\begin{aligned} D_x \cosh u &= \sinh u \frac{du}{dx} \\ D_x \sinh u &= \cosh u \frac{du}{dx} \\ D_x \sinh^{-1} u &= \frac{1}{\sqrt{1+u^2}} \frac{du}{dx} \\ D_x \cosh^{-1} u &= \frac{1}{\sqrt{u^2-1}} \frac{du}{dx} \end{aligned}$$

 $\xrightarrow{\text{FTC}}$ 
INTEGRALS

$$\begin{aligned} \int \sinh u du &= \cosh u + C \\ \int \cosh u du &= \sinh u + C \\ \int \frac{du}{\sqrt{a^2+u^2}} &\stackrel{a>0}{=} \sinh^{-1} \left( \frac{u}{a} \right) + C \\ \int \frac{du}{\sqrt{u^2-a^2}} &\stackrel{u>a>0}{=} \cosh^{-1} \left( \frac{u}{a} \right) + C \end{aligned}$$

<sup>1</sup>Hyperbolic Trig Functions are covered in §7.3, pages 439–447. You not have to memorize these Hyperbolic Trig formulas but should be able to use if given them. They are very important in, e.g., Mechanical/Civil Engineering.

## Basic Integral Formulas

<u>DERIVATIVES</u>	$\xrightarrow{\text{FTC}}$	<u>INTEGRALS</u>
$D_x u^n = nu^{n-1} \frac{du}{dx}$		$\int u^n du \stackrel{n \neq -1}{=} \frac{u^{n+1}}{n+1} + C$
$D_x e^u = e^u \frac{du}{dx}$		$\int e^u du = e^u + C$
$D_x \ln u  \stackrel{u \neq 0}{=} \frac{1}{u} \frac{du}{dx}$		$\int \frac{du}{u} \stackrel{u \neq 0}{=} \ln u  + C$
$D_x a^u = a^u \ln a \frac{du}{dx}$	$0 < a \neq 1$	$\int a^u du = \frac{a^u}{\ln a} + C$
$D_x \log_a u  \stackrel{u \neq 0}{=} \frac{1}{u} \frac{1}{\ln a} \frac{du}{dx}$	$0 < a \neq 1$	
$D_x \sin u = \cos u \frac{du}{dx}$		$\int \cos u du = \sin u + C$
$D_x \tan u = \sec^2 u \frac{du}{dx}$		$\int \sec^2 u du = \tan u + C$
$D_x \sec u = \sec u \tan u \frac{du}{dx}$		$\int \sec u \tan u du = \sec u + C$
$D_x \cos u = -\sin u \frac{du}{dx}$		$\int \sin u du = -\cos u + C$
$D_x \cot u = -\csc^2 u \frac{du}{dx}$		$\int \csc^2 u du = -\cot u + C$
$D_x \csc u = -\csc u \cot u \frac{du}{dx}$		$\int \csc u \cot u du = -\csc u + C$
$D_x \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} = -D_x \cos^{-1} u$		$\int \frac{du}{\sqrt{a^2-u^2}} \stackrel{a \geq 0}{=} \sin^{-1} \frac{u}{a} + C$
$D_x \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx} = -D_x \cot^{-1} u$		$\int \frac{du}{a^2+u^2} \stackrel{a \geq 0}{=} \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
$D_x \sec^{-1} u = \frac{1}{ u \sqrt{u^2-1}} \frac{du}{dx} = -D_x \csc^{-1} u$		$\int \frac{du}{u\sqrt{u^2-a^2}} \stackrel{a \geq 0}{=} \frac{1}{a} \sec^{-1} \frac{ u }{a} + C$

MORE INTEGRALS

$$\begin{aligned}
 \int \tan u du &= -\ln|\cos u| + C &= \ln|\sec u| + C \\
 \int \cot u du &= \ln|\sin u| + C &= -\ln|\csc u| + C \\
 \int \sec u du &= \ln|\sec u + \tan u| + C &= -\ln|\sec u - \tan u| + C \\
 \int \csc u du &= -\ln|\csc u + \cot u| + C &= \ln|\csc u - \cot u| + C
 \end{aligned}$$

## Integration by Parts

$$\int u \, dv = uv - \int v \, du$$

Key Ideas in Integration by Parts.

- For  $\int x^n f(x) \, dx$  where  $\int f(x) \, dx$  is easy, try  $u = x^n$  and  $dv = f(x) \, dx$ . (Note that then  $v = \int dv = \int f(x) \, dx$ .) This often reduces  $x^n$  to  $x^{n-1}$ .
- For  $\int f(x) \, dx$ : if the integrand  $f(x)$  is easy to differentiate but hard to integrate, then try letting  $u = f(x)$  and so  $dv = dx$ .
- *Bring to the other side* (i.e., *loops*) method.
- Creatively look for a  $dv$  that is easy to integrate (since  $v = \int dv$ ).
- None of the others. So what did you learn from this type of problem?

## Trig Identities useful for Integration

Half-Angle Formulas:  $\cos^2 x = \frac{1 + \cos(2x)}{2}$        $\sin^2 x = \frac{1 - \cos(2x)}{2}$

Double-Angle Formulas:  $\cos(2x) = \cos^2 x - \sin^2 x$        $\sin(2x) = 2 \sin x \cos x$

Add./Subst. Formulas:<sup>2</sup>  $\cos(s+t) = \cos s \cos t - \sin s \sin t$  &  $\sin(s+t) = \sin s \cos t + \cos s \sin t$   
 $\cos(s-t) = \cos s \cos t + \sin s \sin t$  &  $\sin(s-t) = \sin s \cos t - \cos s \sin t$

## Trig Substitution

IF INTEGRAND INVOLVES

$$a^2 - u^2$$

$$a^2 + u^2$$

$$u^2 - a^2$$

THEN MAKE THE SUBSTITUTION

$$u = a \sin \theta \iff \theta = \sin^{-1} \frac{u}{a}$$

$$u = a \tan \theta \iff \theta = \tan^{-1} \frac{u}{a}$$

$$u = a \sec \theta \iff \theta = \sec^{-1} \frac{u}{a}$$

RESTRICTION ON  $\theta$

$$\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$\frac{-\pi}{2} < \theta < \frac{\pi}{2}$$

$$0 \leq \theta \leq \pi, \theta \neq \frac{\pi}{2}$$

Space for your personal notes.

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<sup>2</sup>You do not have to memorize the Addition/Subtraction Formulas but should be able to use them if given them.