This quiz reviews some basic trigonometry needed in Calculus II. You may work this quiz either:

(1) on your own paper, in which case, box your answer and show your work
(2) on this paper, in which case, put your answer in the provided box and justify your answer by showing your work beneath the box.

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1. We know that

$$1 \qquad \& \qquad \cos\theta \qquad \& \qquad \sin\theta$$

satisfies the well-known equation

$$\cos^2\theta \ + \ \sin^2\theta \ = \ 1 \ . \tag{1}$$

DERIVE a similar equation (which you will need to know) relating

$$1 \qquad \& \qquad \tan\theta \qquad \& \qquad \text{(one other trigometric function)}(\theta) \ .$$

ANSWER:[1] $\boxed{1 + \tan^2\theta = \sec^2\theta}$

$$\cos^2\theta + \sin^2\theta = 1 \quad \Longrightarrow \quad \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \quad \Longrightarrow \quad 1 + \tan^2\theta = \sec^2\theta$$

2. Express $\arctan\sqrt{3}$ in radians. ANSWER:[2] $\arctan\sqrt{3} = \boxed{\frac{\pi}{3}}$

$$\left[\arctan\sqrt{3} = \theta\right] \Longleftrightarrow \left[\sqrt{3} = \tan\theta \quad \text{and} \quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}\right] \Longleftrightarrow \left[\frac{\sqrt{3}/2}{1/2} = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}\right]$$

Now see the unit circle on the last page.

3. Express $\arctan\left(^-\sqrt{3}\right)$ in radians. ANSWER:[2] $\arctan\left(^-\sqrt{3}\right) = \boxed{-\frac{\pi}{3}}$

$$\left[\arctan\left(^-\sqrt{3}\right) = \theta\right] \Longleftrightarrow \left[-\sqrt{3} = \tan\theta \text{ and } -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}\right] \Longleftrightarrow \left[-\frac{\sqrt{3}/2}{1/2} = \frac{\sin\theta}{\cos\theta} \text{ and } -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}\right]$$

Now see the unit circle on the last page.

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[1] Justify your answer beneath the box. To Derive $\neq$ to look up in a book. Start with the well-known equation in (1) and perform a few algebraic steps to quickly arrive at an equation involving tan. Would you rather memorize this need-to-know equation or just remember how to quickly derive it from the well-known equation (1)? Just for fun: in this problem replace tan with cot and give it a try.

[2] Justify your answer beneath the box, e.g., a properly marked reference triangle or an explanation of how you see it from a properly marked unit circle.

4. Let $x = 5\sec\theta$ and $0 < \theta < \frac{\pi}{2}$.
Without using inverse trigonometric functions, express $\tan\theta$ as a function of x.

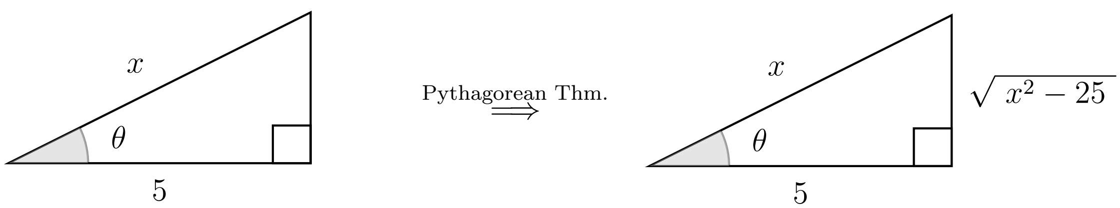
ANSWER:[3] $\tan\theta = \boxed{\dfrac{\sqrt{x^2-25}}{5}}$

<u>**4&5** Way #1</u>. Note

$$x = 5\sec\theta \quad\Longrightarrow\quad \frac{x}{5} = \sec\theta \overset{\text{note}}{=} \frac{1}{\cos\theta}\ .$$

So if $0 < \theta < \frac{\pi}{2}$, then from the below reference triangles we infer

$$\frac{x}{5} = \frac{\text{hyp}}{\text{adj}} \quad\Longrightarrow\quad \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{x^2-25}}{5}\ .$$



Thus $\tan\theta = \pm\frac{\sqrt{x^2-25}}{5}$ and we just now need to determine whether to take the plus or the minus:

$$\tan\theta \ = \begin{cases} +\frac{\sqrt{x^2-25}}{5} & \text{if } \tan\theta \geq 0 \text{ , which is the case for } 0 < \theta < \frac{\pi}{2} \\ -\frac{\sqrt{x^2-25}}{5} & \text{if } \tan\theta \leq 0 \text{ , which is the case for } \frac{\pi}{2} < \theta < \pi\ . \end{cases} \tag{2}$$

5. Let $x = 5\sec\theta$ and $\frac{\pi}{2} < \theta < \pi$.
Without using inverse trigonometric functions, express $\tan\theta$ as a function of x.

ANSWER:[3] $\tan\theta = \boxed{-\dfrac{\sqrt{x^2-25}}{5}}$

<u>**4&5** Way #2</u>. Recall that

$$\cos^2\theta + \sin^2\theta = 1 \quad\Longrightarrow\quad \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \quad\Longrightarrow\quad 1 + \tan^2\theta = \sec^2\theta\ .$$

We are given that $\sec\theta = \frac{x}{5}$ and so

$$\tan^2\theta = \sec^2\theta - 1 = \frac{x^2}{5^2} - 1 = \frac{x^2-25}{5^2} = \left(\frac{\pm\sqrt{x^2-25}}{5}\right)^2\ .$$

Thus $\tan\theta = \pm\frac{\sqrt{x^5-25}}{5}$ and we just now need to determine whether to take the plus or the minus and so we proceed as we did in Way # 1 in (2) :

[3]Justify your work either by some algebra or by a properly marked unit circle/reference triangle, along with a brief explanation of what you are thinking.

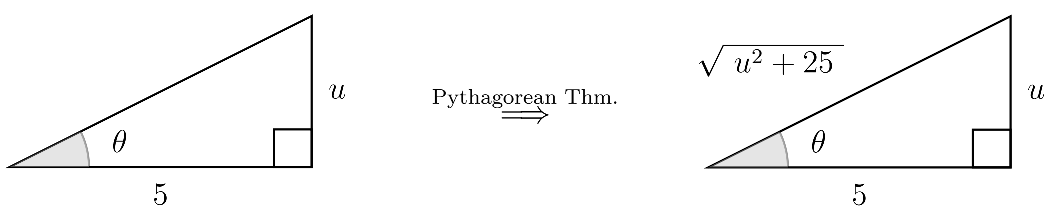
6. Let $u = 5\tan\theta$. Without using inverse trigonometric functions, fill out the below chart to express $\sin\theta$ as a function of u.

Answer : [3] $$\sin\theta = \begin{cases} \boxed{+\dfrac{u}{\sqrt{u^2+25}}} & \text{;if } 0 \le \theta < \frac{\pi}{2} \\[2ex] \boxed{-\dfrac{u}{\sqrt{u^2+25}}} & \text{;if } \frac{\pi}{2} < \theta < \pi \\[2ex] \boxed{-\dfrac{u}{\sqrt{u^2+25}}} & \text{;if } \pi \le \theta < \frac{3\pi}{2} \\[2ex] \boxed{+\dfrac{u}{\sqrt{u^2+25}}} & \text{;if } \frac{3\pi}{2} < \theta < 2\pi. \end{cases}$$

Hint: For each quadant, compare the sign (i.e., positive or negative) of $\sin\theta$ with the sign of $\tan\theta$.

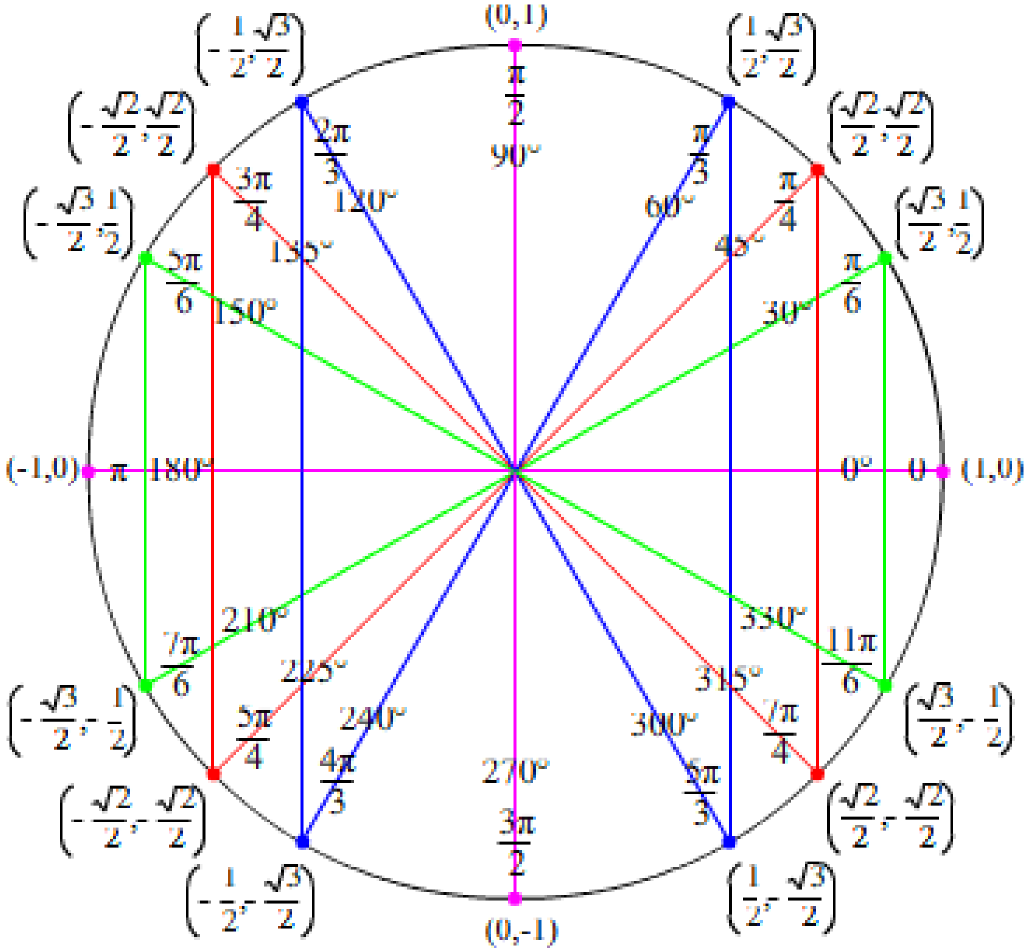
<u>Justification</u>: If $0 < \theta < \frac{\pi}{2}$, then from the below reference triangles we infer

$$\frac{u}{5} = \tan\theta = \frac{\text{opp}}{\text{adj}} \quad \Longrightarrow \quad \sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{u}{\sqrt{u^2+25}}\ .$$



Thus $\sin\theta = \pm\frac{u}{\sqrt{u^2+25}}$ and we just now need to determine whether to take the plus or the minus. The choice of $\pm$ follows from the fact that $\sin\theta$ and u have the same sign in the 1st and 4th quadants while they have the opposite signs in the 2nd and 3rd quadants.

Unit Circle


(0,1)
(1,0)
(0,-1)
(-1,0)
0°
90°
180°
270°
30°
45°
60°
120°
135°
150°
210°
225°
240°
300°
315°
330°
π
0