Definition. The *N-th order Taylor polynominal* os $y = f(x)$ at x_0 is the polynomial

$$P_N(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \ldots \frac{f^{(N)}(x_0)}{N!}(x - x_0)^N$$

which can also be written as

$$P_N(x) = \frac{f^{(0)}(x_0)}{0!} + \frac{f^{(1)}(x_0)}{1!}(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \ldots \frac{f^{(N)}(x_0)}{N!}(x - x_0)^N .$$

Here we are assuming that the derivatives $y = f^{(n)}(x)$ exist for each x in the interval I and for each $n \in \mathbb{N}$.

Below, find some Nth-order Taylor Polynomials of the given function at the given center.

Example 1. for the function $f(x) = \frac{1}{1-x}$ center at $x_0 = 0$

Helpful Table for Example 1			
n	$f^{(n)}(x)$	$f^{(n)}(x_0) \overset{\text{here}}{=} f^{(n)}(0)$	$c_n \overset{\text{def}}{=} \frac{f^{(n)}(x_0)}{n!} \overset{\text{here}}{=} \frac{f^{(n)}(0)}{n!}$
0	$(1-x)^{-1}$	1	$\frac{1}{0!} = \frac{1}{1} = 1$
1			
2			
3			
4			
5			
6			

Example 2. for the function $f(x) = \sin x$ center at $x_0 = \pi$

Helpful Table for Example 1			
n	$f^{(n)}(x)$	$f^{(n)}(x_0) \overset{\text{here}}{=} f^{(n)}(\pi)$	$c_n \overset{\text{def}}{=} \frac{f^{(n)}(x_0)}{n!} \overset{\text{here}}{=} \frac{f^{(n)}(\pi)}{n!}$
0	$\sin x$	0	$\frac{0}{0!} = \frac{0}{1} = 0$
1	$\cos x$		
2	$-\sin x$		
3	$-\cos x$		
4	$\sin x$ (starts to repeat/cycle)		

Example 3. Next we consider the function $f(x) = \ln(1 + x)$ about the center $x_0 = 0$.

Helpful Table for Example 3			
n	$f^{(n)}(x)$	$f^{(n)}(x_0) \overset{\text{here}}{=} f^{(n)}(0)$	$c_n \overset{\text{def}}{=} \frac{f^{(n)}(x_0)}{n!} \overset{\text{here}}{=} \frac{f^{(n)}(0)}{n!}$
0			
1			
2			
3			
4			
5			
6			
$\vdots$			
n			