SERIOUS SERIES' PROBLEMS

Determine whether each series converges absolutely, converges conditionally, or is divergent. *Justify your answer.*

(1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(2) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(3) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

(4) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$

(5) $\sum_{n=0}^{\infty} \frac{n+1}{n!}$

(6) $\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

(7) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(8) $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{2n^2+n-1}$

(9) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

(10) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$

(11) $\sum_{n=1}^{\infty} \frac{1}{(3n-2)^{n+(1/2)}}$

(12) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$

(13) $\sum_{n=1}^{\infty} \frac{\ln(n!)}{n^3}$

(14) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n n!}{(2n)!}$

(15) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)^n$

SERIOUS SERIES' PROBLEMS – HINTS

Below are just HINTS, without the needed justifications.

Recall that often there is more than one way to determine the behavior of a series.

abs. conv. – absolutely convergent
cond. conv. – conditionally convergent
divg. – divergent

AST – Alternating Series Test
CT – Comparison Test
LCT – Limit Comparison Test

(1) divg. – p-series with $p = \frac{1}{2}$
(2) cond. conv. – AST & p-series with $p = \frac{1}{2}$
(3) cond. conv. – AST & CT to $\frac{1}{n+1}$
(4) abs. conv. – LCT to $\frac{1}{n^2}$
(5) abs. conv. – ratio test $\rho = 0$
(6) abs. conv. – ratio test $\rho = 0$
(7) abs. conv. – integral test
(8) divg. – n^{th}-term test for divergence
(9) abs. conv. – root test $\rho = 0$
(10) abs. conv. – LCT to $\left(\frac{1}{n}\right)^{\frac{3}{2}}$
(11) abs. conv. – CT to $\dfrac{1}{(3n-2)^n}$ & do the root test to $\dfrac{1}{(3n-2)^n}$
(12) abs. conv. – CT to $\frac{2}{n^2}$. note that $|\arctan n| \leq \frac{\pi}{2}$.
(13) abs. conv. – CT to $\left(\frac{1}{n}\right)^{\frac{3}{2}}$. note that
$\ln(n!) = \ln(1 \cdot 2 \cdots n) = \ln 1 + \ln 2 + \ldots \ln n \ \leq n \ \ln n$
and so for big n
$$\frac{\ln(n!)}{n^3} \leq \frac{n \ \ln n}{n^3} = \frac{\ln n}{n^2} \leq \frac{n^{\frac{1}{2}}}{n^2} = \frac{1}{n^{\frac{3}{2}}}$$
(14) abs. conv. – ratio test $\rho = 0$
(15) divg. – n^{th}-term test for divergence. note that
$$\left(\frac{n}{n+1}\right)^n = \left[\left(\frac{n+1}{n}\right)^n\right]^{-1} = \left[\left(1 + \frac{1}{n}\right)^n\right]^{-1} \to \left[e^1\right]^{-1} = e^{-1} \neq 0.$$