Parametric Curves

In this part, fill in the 4 boxes. Consider the curve $\mathcal{C}$ parameterized by

$$x = x(t)$$
$$y = y(t)$$

for $a \le t \le b$.

1) Express $\frac{dy}{dx}$ in terms of derivatives with respect to t. Answer: $\frac{dy}{dx} = \boxed{\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}}$

2) The tangent line to $\mathcal{C}$ when $t = t_0$ is $y = mx + b$ where m is $\boxed{\frac{dy}{dx}}$ evaluated at $t = t_0$.

3) Express $\frac{d^2y}{dx^2}$ using derivatives with respect to t. Answer: $\frac{d^2y}{dx^2} = \boxed{\dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}}$

4) The arc length of $\mathcal{C}$, expressed as on integral with respect to t, is

$$\text{Arc Length} = \boxed{\int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \; dt}$$

Polar Coordinates

- ▶. Here, CC stands for *Cartresian coordinates* while PC stands for *polar coordinates.*
- A point with PC (r, θ) also has PC $\left(\boxed{r}, \theta + 2\pi\right)$ as well as $\left(\boxed{-r}, \theta + \pi\right)$.
- A point $P \in \mathbb{R}^2$ with CC (x, y) and PC (r, θ) satisfies the following.

$$x = \boxed{r\ \cos\theta} \quad \& \quad y = \boxed{r\ \sin\theta} \quad \& \quad r^2 = \boxed{x^2 + y^2} \quad \& \quad \boxed{\tan\theta} = \begin{cases} \frac{y}{x} & \text{if } x \neq 0 \\ \text{DNE} & \text{if } x = 0 \end{cases}.$$

- The period of $f(\theta) = \cos(k\theta)$ and of $f(\theta) = \sin(k\theta)$ is $\boxed{\frac{2\pi}{k}}$.

  To sketch these graphs, we divide the period by $\boxed{4}$ and make *the chart*,

  in order to detect the $\boxed{\text{max/min/zero's of the function } r = f(\theta)}$.
- Now consider a sufficiently *nice* function $r = f(\theta)$ which determines a curve in the plane. The the area bounded by polar curves $r = f(\theta)$ and the rays $\theta = \alpha$ and $\theta = \beta$ is

$$\text{Area} = \int_{\theta=\alpha}^{\theta=\beta} \boxed{\frac{1}{2}\,[f(\theta)]^2} \; d\theta \,.$$

The arc length of the polar curves $r = f(\theta)$ is

$$\text{Arc Length} = \int_{\theta=\alpha}^{\theta=\beta} \boxed{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} \; d\theta \,.$$