

Parametric Curves

In this part, fill in the 4 boxes. Consider the curve \mathcal{C} parameterized by

$$\begin{aligned}x &= x(t) \\ y &= y(t)\end{aligned}$$

for $a \leq t \leq b$.

1) Express $\frac{dy}{dx}$ in terms of derivatives with respect to t . Answer: $\frac{dy}{dx} =$

2) The tangent line to \mathcal{C} when $t = t_0$ is $y = mx + b$ where m is

evaluated at $t = t_0$.

3) Express $\frac{d^2y}{dx^2}$ using derivatives with respect to t . Answer: $\frac{d^2y}{dx^2} =$

4) The arc length of \mathcal{C} , expressed as an integral with respect to t , is

Arc Length =

Polar Coordinates

►. Here, CC stands for *Cartesian coordinates* while PC stands for *polar coordinates*.

- A point with PC (r, θ) also has PC $(\square, \theta + 2\pi)$ as well as $(\square, \theta + \pi)$.
- A point $P \in \mathbb{R}^2$ with CC (x, y) and PC (r, θ) satisfies the following.

$$x = \square \quad \& \quad y = \square \quad \& \quad r^2 = \square \quad \& \quad \square = \begin{cases} \frac{y}{x} & \text{if } x \neq 0 \\ \text{DNE} & \text{if } x = 0. \end{cases}$$

- The period of $f(\theta) = \cos(k\theta)$ and of $f(\theta) = \sin(k\theta)$ is \square .

To sketch these graphs, we divide the period by \square and make *the chart*,

in order to detect the

- Now consider a sufficiently *nice* function $r = f(\theta)$ which determines a curve in the plane. The the area bounded by polar curves $r = f(\theta)$ and the rays $\theta = \alpha$ and $\theta = \beta$ is

$$\text{Area} = \int_{\theta=\alpha}^{\theta=\beta} \square \, d\theta .$$

The arc length of the polar curves $r = f(\theta)$ is

$$\text{Arc Length} = \int_{\theta=\alpha}^{\theta=\beta} \square \, d\theta .$$