A. Power Series

Consider the (formal) power series

$$h(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n , \tag{1}$$

with radius of convergence $R \in [0, \infty]$.
(Here $x_0 \in \mathbb{R}$ is fixed and $\{a_n\}_{n=0}^{\infty}$ is a fixed sequence of real numbers.)
Without any other further information on $\{a_n\}_{n=0}^{\infty}$, answer the following questions.

A.1. The choices for nest 4 boxes are: AC, CC, DIVG, anything. Here,

AC stands for: *is always absolutely convergent*
CC stands for: *is always conditionally convergent*
DIVG stands for: *is always divergent*
anything stands for: *can do anything, i.e., there are examples showing that it can be AC, CC, or DIVG.*

(1) At the center $x = x_0$, the power series in (1) $\boxed{}$.
(2) For $x \in \mathbb{R}$ such that $|x - x_0| < R$, the power series in (1) $\boxed{}$.
(3) For $x \in \mathbb{R}$ such that $|x - x_0| > R$, the power series in (1) $\boxed{}$.
(4) If $R > 0$, then for the endpoints $x = x_0 \pm R$, the power series in (1) $\boxed{}$.

A.2. Now let $R > 0$ and fill-in the 7 boxes.
Consider the function $y = h(x)$ defined by the power series in (1).

(a) The function $y = h(x)$ is always differentiable on the interval $\boxed{}$ (make this interval as large as it can be, but still keeping the statement true). Also, on this interval

$$h'(x) = \sum_{n=\boxed{}}^{\infty} \boxed{} . \tag{2}$$

What can you say about the radius of convergence of the power series in (2)?
$\boxed{}$.

(b) The function $y = h(x)$ always has an antiderivative on the interval $\boxed{}$ (make this interval as large as it can be, but still keeping the statement true). Futhermore, if α and β are in this interval, then

$$\int_{x=\alpha}^{x=\beta} h(x)\, dx = \sum_{n=\boxed{}}^{\infty} \boxed{} \Bigg|_{\mathbf{x}=\alpha}^{\mathbf{x}=\beta} .$$

B. Taylor/Maclaurin Polynomials and Series

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0.
Let $y = P_N(x)$ be the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0.
Let $y = R_N(x)$ be the N^{th}-order Taylor remainder of $y = f(x)$ about x_0.
Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0.
Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0.

B.1. The formula for c_n is

$$c_n = \boxed{}$$

B.2. In open form (i.e., with ... and without a $\sum$-sign)

$P_N(x) =$ [ ]

B.3. In closed form (i.e., with a $\sum$-sign and without ...)

$P_N(x) =$ [ ]

B.4. In open form (i.e., with ... and without a $\sum$-sign)

$P_\infty(x) =$ [ ]

B.5. In closed form (i.e., with a $\sum$-sign and without ...)

$P_\infty(x) =$ [ ]

B.6. We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$R_N(x) =$ [ ] for some c between [ ] and [ ].

B.7. A Maclaurin series is a Taylor series with the center specifically specified as $x_0 =$ [ ].

C. Commonly Used Taylor Series

Here, *expansion* refers to the power series expansion that is the Maclaurin series.

C.1. An expansion for $y = e^x$ is [ ], which is valid precisely when $x \in$ [ ].

C.2. An expansion for $y = \cos x$ is [ ], which is valid precisely when $x \in$ [ ].

C.3. An expansion for $y = \sin x$ is [ ], which is valid precisely when $x \in$ [ ].

C.4. An expansion for $y = \frac{1}{1-x}$ is [ ], which is valid precisely when $x \in$ [ ].

C.5. An expansion for $y = \ln(1+x)$ is [ ], which is valid precisely when $x \in$ [ ].

C.6. An expansion for $y = \arctan x$ is [ ], which is valid precisely when $x \in$ [ ].