0. Fill-in-the boxes.

Power Series Consider the (formal) power series

$$h(x) \;=\; \sum_{n=0}^{\infty} a_n \, (x - x_0)^n \ , \tag{1}$$

with radius of convergence $R \in [0, \infty]$.

(Here $x_0 \in \mathbb{R}$ is fixed and $\{a_n\}_{n=0}^{\infty}$ is a fixed sequence of real numbers.)

Without any other further information on $\{a_n\}_{n=0}^{\infty}$, answer the following questions.

0.1. First let $0 < R < \infty$. The largest set of x's for which we know that the power series in (1) is:

(a) absolutely convergent is ______________________________

(b) divergent is ______________________________ .

What can you say about the convergence of the power series in (1) when $x = x_0 + R$ or $x = x_0 - R$?

______________________________ .

0.2. Now let $R = \infty$. The largest set of x's for which we know that the power series in (1) is:

(a) absolutely convergent is ______________________________

(b) divergent is ______________________________ .

0.3. Now let $R = 0$. The largest set of x's for which we know that the power series in (1) is:

(a) absolutely convergent is ______________________________

(b) divergent is ______________________________ .

0.4. Now let $R > 0$ and fill-in the 5 boxes.

Consider the function $y = h(x)$ defined by the power series in (1).

(a) The function $y = h(x)$ is always differentiable on the interval $\boxed{}$ (make this interval as large as it can be, but still keeping the statement true). Furthermore, on this interval

$$h'(x) \;=\; \sum_{n=1}^{\infty} \boxed{} \ . \tag{2}$$

What can you say about the radius of convergence of the power series in (2)? $\boxed{}$.

(b) The function $y = h(x)$ always has an antiderivative on the interval $\boxed{}$ (make this interval as large as it can be, but still keeping the statement true). Futhermore, if α and β are in this interval, then

$$\int_{x=\alpha}^{x=\beta} h(x)\, dx \;=\; \sum_{n=0}^{\infty} \boxed{} \Bigg|_{\mathbf{x}=\alpha}^{\mathbf{x}=\beta} \ .$$