0A. Power Series Consider the (formal) power series

$$h(x) \;=\; \sum_{n=0}^{\infty} a_n \, (x - x_0)^n \;, \tag{1}$$

with radius of convergence $R \in [0, \infty]$.
(Here $x_0 \in \mathbb{R}$ is fixed and $\{a_n\}_{n=0}^{\infty}$ is a fixed sequence of real numbers.)
Without any other further information on $\{a_n\}_{n=0}^{\infty}$, answer the following questions.
If you are having troubles, see the §10.7 handout *Operations on Power Series* at
http://people.math.sc.edu/girardi/m142/handouts/OperationsOnPowerSeries.pdf .

- First let $0 < R < \infty$. The largest set of x's for which we know that the power series in (1) is:

  (a) absolutely convergent is ____________________

  (b) divergent is ____________________ .

  What can you say about the convergence of the power series in (1) when $x = x_0 + R$ or $x = x_0 - R$?

  ____________________ .

- Now let $R = \infty$. The largest set of x's for which we know that the power series in (1) is:

  (a) absolutely convergent is ____________________

  (b) divergent is ____________________ .

- Now let $R = 0$. The largest set of x's for which we know that the power series in (1) is:

  (a) absolutely convergent is ____________________

  (b) divergent is ____________________ .

- Now let $R > 0$ and fill-in the 5 boxes.
  Consider the function $y = h(x)$ defined by the power series in (1).

  (a) The function $y = h(x)$ is always differentiable on the interval [ ] (make this interval as large as it can be, but still keeping the statement true). Furthermore, on this interval

  $$h'(x) \;=\; \sum_{n=1}^{\infty} \boxed{} \;. \tag{2}$$

  What can you say about the radius of convergence of the power series in (2)? [ ].

  (b) The function $y = h(x)$ always has an antiderivative on the interval [ ] (make this interval as large as it can be, but still keeping the statement true). Futhermore, if α and β are in this interval, then

  $$\int_{x=\alpha}^{x=\beta} h(x)\, dx \;=\; \sum_{n=0}^{\infty} \boxed{} \Bigg|_{\mathbf{x}=\alpha}^{\mathbf{x}=\beta} \;.$$

0B. Taylor/Maclaurin Polynomials and Series. Fill-in the boxes.
If you are having troubles, see the class handout for §10.8 at
http://people.math.sc.edu/girardi/m142/handouts/16sTaylorPoly.pdf
and the handout for §10.8/9/10 at
http://people.math.sc.edu/girardi/m142/handouts/16sTaylorSeries.pdf .

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0.

Let $y = P_N(x)$ be the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0.

Let $y = R_N(x)$ be the N^{th}-order Taylor remainder of $y = f(x)$ about x_0.

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0.

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0.

a. In open form (i.e., with "…" notation and without a $\sum$-sign)

$P_N(x) =$ [ ]

b. In closed form (i.e., with a $\sum$-sign and without "…" notation)

$P_N(x) =$ [ ]

c. In open form (i.e., with "…" notation and without a $\sum$-sign)

$P_\infty(x) =$ [ ]

d. In closed form (i.e., with a $\sum$-sign and without "…" notation)

$P_\infty(x) =$ [ ]

e. The formula for c_n is

$c_n =$ [ ]

f. We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$R_N(x) =$ [ ] for some c between [ ] and [ ].

g. A Maclaurin series is a Taylor series with the center specifically specified as $x_0 =$ [ ].