

0. Fill-in-the boxes. Below, $a, b, c \in \mathbb{R}$ with $a < c < b$.

0.1. If $f: [0, \infty) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_0^\infty f(x) dx$ by

$$\int_0^\infty f(x) dx = \boxed{\lim_{t \rightarrow \infty} \int_0^t f(x) dx}.$$

0.2. If $f: (-\infty, 0] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^0 f(x) dx$ by

$$\int_{-\infty}^0 f(x) dx = \boxed{\lim_{t \rightarrow -\infty} \int_t^0 f(x) dx}.$$

0.3. If $f: (-\infty, \infty) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^\infty f(x) dx$ by

$$\int_{-\infty}^\infty f(x) dx = \boxed{\left[\lim_{t \rightarrow -\infty} \int_t^0 f(x) dx \right] + \left[\lim_{s \rightarrow \infty} \int_0^s f(x) dx \right]}.$$

0.4. If $f: (a, b] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{\lim_{t \rightarrow a^+} \int_t^b f(x) dx}.$$

0.5. If $f: [a, b) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{\lim_{t \rightarrow b^-} \int_a^t f(x) dx}.$$

0.6. If $f: [a, c) \cup (c, b] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{\left[\lim_{t \rightarrow c^-} \int_a^t f(x) dx \right] + \left[\lim_{s \rightarrow c^+} \int_s^b f(x) dx \right]}.$$

0.7. An improper integral as above *converges* precisely when

each of the limits involves converges to a **finite** number.

0.8. An improper integral as above *diverges* precisely when

the improper integral does not converge.