0. Fill-in-the boxes. Below, $a, b, c \in \mathbb{R}$ with $a < c < b$.

0.1. If $f\colon [0, \infty) \to \mathbb{R}$ is continuous, then we define the improper integral $\int_0^\infty f(x)\, dx$ by

$$\int_0^\infty f(x)\, dx = \boxed{}\ .$$

0.2. If $f\colon (-\infty, 0] \to \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^0 f(x)\, dx$ by

$$\int_{-\infty}^0 f(x)\, dx = \boxed{}\ .$$

0.3. If $f\colon (-\infty, \infty) \to \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^\infty f(x)\, dx$ by

$$\int_{-\infty}^\infty f(x)\, dx = \boxed{}\ .$$

0.4. If $f\colon (a, b] \to \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x)\, dx$ by

$$\int_a^b f(x)\, dx = \boxed{}\ .$$

0.5. If $f\colon [a, b) \to \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x)\, dx$ by

$$\int_a^b f(x)\, dx = \boxed{}\ .$$

0.6. If $f\colon [a, c) \cup (c, b] \to \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x)\, dx$ by

$$\int_a^b f(x)\, dx = \boxed{}\ .$$

0.7. An improper integral as above *converges* precisely when

$$\boxed{}$$

0.8. An improper integral as above *diverges* precisely when

$$\boxed{}$$

0. Fill-in-the boxes. Below, $a, b, c \in \mathbb{R}$ with $a < c < b$.

0.1. If $f\colon [0,\infty) \to \mathbb{R}$ is continuous, then we define the improper integral $\int_0^\infty f(x)\,dx$ by

$$\int_0^\infty f(x)\,dx = \boxed{\lim_{t\to\infty} \int_0^t f(x)\,dx}.$$

0.2. If $f\colon (-\infty,0] \to \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^0 f(x)\,dx$ by

$$\int_{-\infty}^0 f(x)\,dx = \boxed{\lim_{t\to-\infty} \int_t^0 f(x)\,dx}.$$

0.3. If $f\colon (-\infty,\infty) \to \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^\infty f(x)\,dx$ by

$$\int_{-\infty}^\infty f(x)\,dx = \boxed{\left[\lim_{t\to-\infty} \int_t^0 f(x)\,dx\right] + \left[\lim_{s\to\infty} \int_0^s f(x)\,dx\right]}.$$

0.4. If $f\colon (a,b] \to \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x)\,dx$ by

$$\int_a^b f(x)\,dx = \boxed{\lim_{t\to a^+} \int_t^b f(x)\,dx}.$$

0.5. If $f\colon [a,b) \to \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x)\,dx$ by

$$\int_a^b f(x)\,dx = \boxed{\lim_{t\to b^-} \int_a^t f(x)\,dx}.$$

0.6. If $f\colon [a,c) \cup (c,b] \to \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x)\,dx$ by

$$\int_a^b f(x)\,dx = \boxed{\left[\lim_{t\to c^-} \int_a^t f(x)\,dx\right] + \left[\lim_{s\to c^+} \int_s^b f(x)\,dx\right]}.$$

0.7. An improper integral as above *converges* precisely when

each of the limits involves converges to a **finite** number.

0.8. An improper integral as above *diverges* precisely when

the improper integral does not converge.