

0. Fill-in-the boxes. Below, $a, b, c \in \mathbb{R}$ with $a < c < b$.

0.1. If $f: [0, \infty) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_0^{\infty} f(x) dx$ by

$$\int_0^{\infty} f(x) dx = \boxed{\phantom{\int_0^{\infty} f(x) dx}}.$$

0.2. If $f: (-\infty, 0] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^0 f(x) dx$ by

$$\int_{-\infty}^0 f(x) dx = \boxed{\phantom{\int_{-\infty}^0 f(x) dx}}.$$

0.3. If $f: (-\infty, \infty) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^{\infty} f(x) dx$ by

$$\int_{-\infty}^{\infty} f(x) dx = \boxed{\phantom{\int_{-\infty}^{\infty} f(x) dx}}.$$

0.4. If $f: (a, b] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{}.$$

0.5. If $f: [a, b) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{}.$$

0.6. If $f: [a, c) \cup (c, b] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{}.$$

0.7. An improper integral as above *converges* precisely when

0.8. An improper integral as above *diverges* precisely when