The Fundamental Theorem of Calculus (FTC) gives that if f is continuous on $[a, b]$, then

$$\int_a^b f(x)\ dx \ = \ F(b) - F(a)$$

where F is any antiderivative of f, i.e., a function such that $F' = f$. Note that to apply the FTC, one needs f to be continuous on the whole interval $[a, b]$. Thus one needs the following 3 conditions.

(1) If $a < c < b$ (in other words, if $c \in (a, b)$), then

$$\lim_{x \to c} f(x) = f(c) \ .$$

(2) The limit of $y = f(x)$ as x approaches a from the right is $f(a)$, which can be expressed

$$\lim_{x \to a^+} f(x) = f(a) \ .$$

(3) The limit of $y = f(x)$ as x approaches b from the left is $f(b)$, which can be expressed

$$\lim_{x \to b^-} f(x) = f(b) \ .$$

To help prepare yourself for the lecture on Improper Integrals, read the section on Improper Integrals from the textbook. Then answer the following questions.

Consider the function

$$f(x) \ = \ \frac{1}{\sqrt{|x - 17|}} \ .$$

Note that the function $y = f(x)$ is not defined at $x = 17$ and is continuous on $\mathbb{R} \setminus \{17\}$[1].

1. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals $\int_a^b f(x)\ dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{18}^{\infty} \frac{1}{\sqrt{|x - 17|}}\ dx \ = \ \lim_{\boxed{t \to \infty}} \int_{x=\boxed{18}}^{x=\boxed{t}} f(x)\ dx \ .$$

2. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals $\int_a^b f(x)\ dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{17}^{18} \frac{1}{\sqrt{|x - 17|}}\ dx \ = \ \lim_{\boxed{t \to 17^+}} \int_{x=\boxed{t}}^{x=\boxed{18}} f(x)\ dx \ .$$

[1] Recall that $\mathbb{R} \setminus \{17\}$ denotes the whole real line *take away* the point 17, i.e., $(-\infty, 17) \cup (17, \infty)$.

3. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals $\int_a^b f(x)\,dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{16}^{17} \frac{1}{\sqrt{|x-17|}}\,dx = \lim_{\boxed{t \to 17^-}} \int_{x=\boxed{16}}^{x=\boxed{t}} f(x)\,dx\ .$$

4. Fill in the 6 boxes as so to express the following integral as a sum of two limits of definite integrals $\int_a^b f(x)\,dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{16}^{18} \frac{1}{\sqrt{|x-17|}}\,dx =$$

$$\lim_{\boxed{t \to 17^-}} \int_{x=\boxed{16}}^{x=\boxed{t}} f(x)\,dx + \lim_{\boxed{t \to 17^+}} \int_{x=\boxed{t}}^{x=\boxed{18}} f(x)\,dx\ .$$

5. Express the following integral as a sum of appropriate limits of definite integrals $\int_a^b f(x)\,dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{16}^{\infty} \frac{1}{\sqrt{|x-17|}}\,dx =$$

$$\lim_{t \to 17^-} \int_{16}^{t} f(x)\,dx + \lim_{t \to 17^+} \int_{t}^{18} f(x)\,dx + \lim_{t \to \infty} \int_{18}^{t} f(x)\,dx\ .$$