

This quiz, which you can do after §8.2, is **DUE** Tuesday February 13 at the **beginning** of class. The Fundamental Theorem of Calculus (FTC) gives that if f is continuous on $[a, b]$, then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

where F is any antiderivative of f , i.e., a function such that $F' = f$. Note that to apply the FTC, one needs f to be continuous on the whole interval $[a, b]$. Thus one needs the following 3 conditions.

(1) If $a < c < b$ (in other words, if $c \in (a, b)$), then

$$\lim_{x \rightarrow c} f(x) = f(c) \quad .$$

(2) The limit of $y = f(x)$ as x approaches a from the right is $f(a)$, which can be expressed

$$\lim_{x \rightarrow a^+} f(x) = f(a) \quad .$$

(3) The limit of $y = f(x)$ as x approaches b from the left is $f(b)$, which can be expressed

$$\lim_{x \rightarrow b^-} f(x) = f(b) \quad .$$

To help prepare yourself for the lecture on Improper Integrals, **read** from the start of §8.8 through the subsection *Improper Integrals with a CAS* (on p. 504–509). Then answer the following questions.

Consider the function

$$f(x) = \frac{1}{\sqrt{|x - 17|}} \quad .$$

Note that the function $y = f(x)$ is not defined at $x = 17$ and is continuous on $\mathbb{R} \setminus \{17\}$ ¹.

1. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals $\int_a^b f(x) \, dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{18}^{\infty} \frac{1}{\sqrt{|x - 17|}} \, dx = \lim_{\boxed{t \rightarrow \infty}} \int_{\boxed{18}}^{\boxed{t}} f(x) \, dx \quad .$$

2. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals $\int_a^b f(x) \, dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{17}^{18} \frac{1}{\sqrt{|x - 17|}} \, dx = \lim_{\boxed{t \rightarrow 17^+}} \int_{\boxed{t}}^{\boxed{18}} f(x) \, dx \quad .$$

¹Recall that $\mathbb{R} \setminus \{17\}$ denotes the whole real line *take away* the point 17, i.e., $(-\infty, 17) \cup (17, \infty)$.

3. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals $\int_a^b f(x) dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{16}^{17} \frac{1}{\sqrt{|x-17|}} dx = \lim_{\boxed{t \rightarrow 17^-}} \int_{\boxed{16}}^{\boxed{t}} f(x) dx .$$

4. Fill in the 6 boxes as so to express the following integral as a sum of two limits of definite integrals $\int_a^b f(x) dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{16}^{18} \frac{1}{\sqrt{|x-17|}} dx = \lim_{\boxed{t \rightarrow 17^-}} \int_{\boxed{16}}^{\boxed{t}} f(x) dx + \lim_{\boxed{s \rightarrow 17^+}} \int_{\boxed{s}}^{\boxed{18}} f(x) dx .$$

5. Express the following integral as a sum of appropriate limits of definite integrals $\int_a^b f(x) dx$ where $y = f(x)$ is continuous on $[a, b]$.

$$\int_{16}^{\infty} \frac{1}{\sqrt{|x-17|}} dx = \lim_{t \rightarrow 17^-} \int_{16}^t f(x) dx + \lim_{s \rightarrow 17^+} \int_s^{18} f(x) dx + \lim_{u \rightarrow \infty} \int_{18}^u f(x) dx .$$