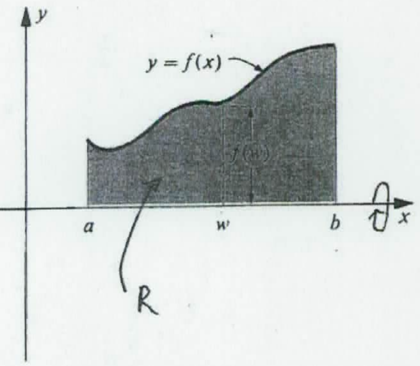
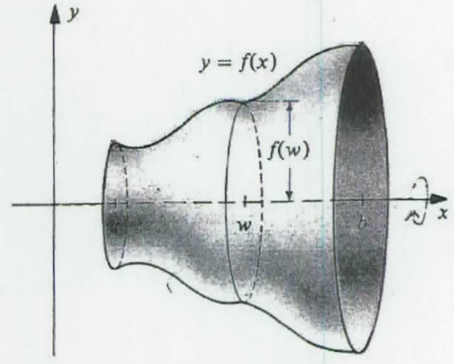
§ 6.2 & 6.3 Volume of Solids of Revolution

(i) Start out with a region R in the xy-plane

(ii) Revolve (i.e. spin) the region R around a horizontal or vertical line to form your "Solid of Revolution"



(i)



(ii)

(iii) Find the volume V of this Solid of Revolution by expressing V as a definite integral and integrating it.

Game Plan

1. partition the appropriate axis (either the x-axis or y-axis)
   If you partition the z-axis, then $V = \int_{z=\#}^{z=\#} (\text{some function of } z)\, dz$.

see (iii) next 2 pages: (i) → 2. form Riemann typical rectangles as if you were looking for the Area of R.

→ 3. revolve the typical rectangles to get "typical elements", here an element will be: disk or washer or shell.

4. Find the volume $\tilde{V}$ of a typical element. $\tilde{V}$ will look like $\tilde{V} = (\text{some function of } z)\, \Delta z$.

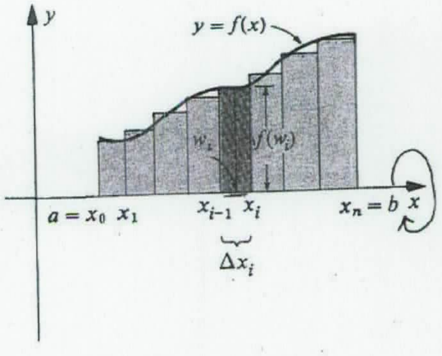
5. Sum the volume of all the typical elements resulting from your partition

6. take the limit as $\Delta z \to 0$ to get

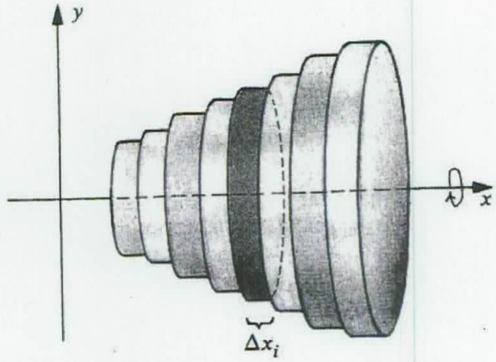
$$V = \int_{z=\#}^{z=\#} (\text{that some function of } z)\, dz.$$

the same

§6.2 Disk Method revolve abt x-axis & partition x-axis



(i)



(ii)

no hole

⇓

disk

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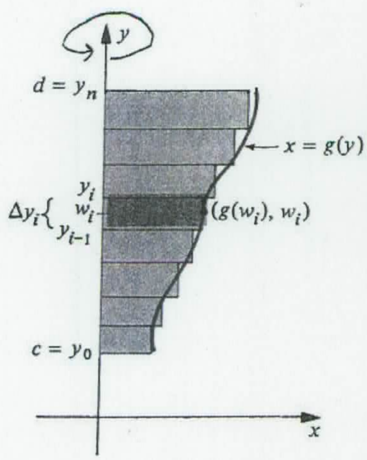
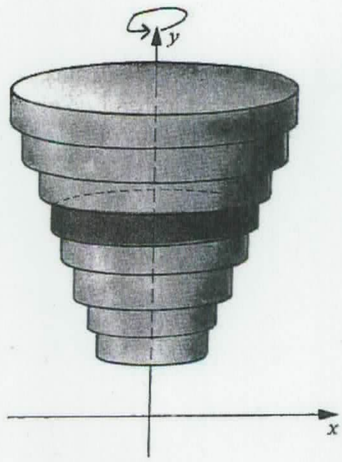
§6.2 Disk Method revolve abt y-axis & partition y-axis



(i)



(ii)

no hole

⇓

disk

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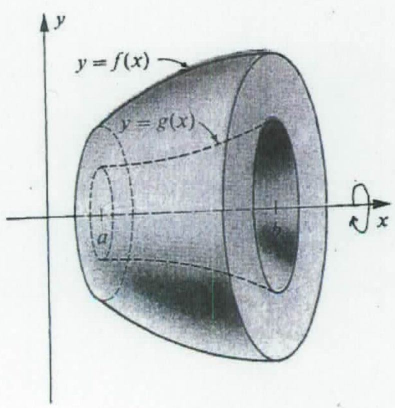
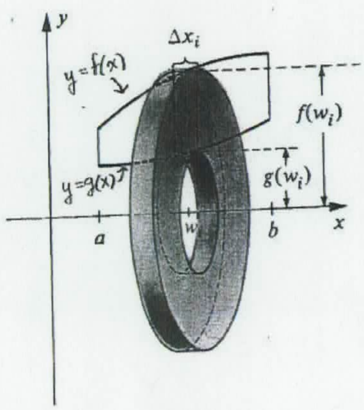
§6.2 Washer Method revolve abt x-axis partitioned x-axis



(i)



(ii)

has hole

⇓

washer

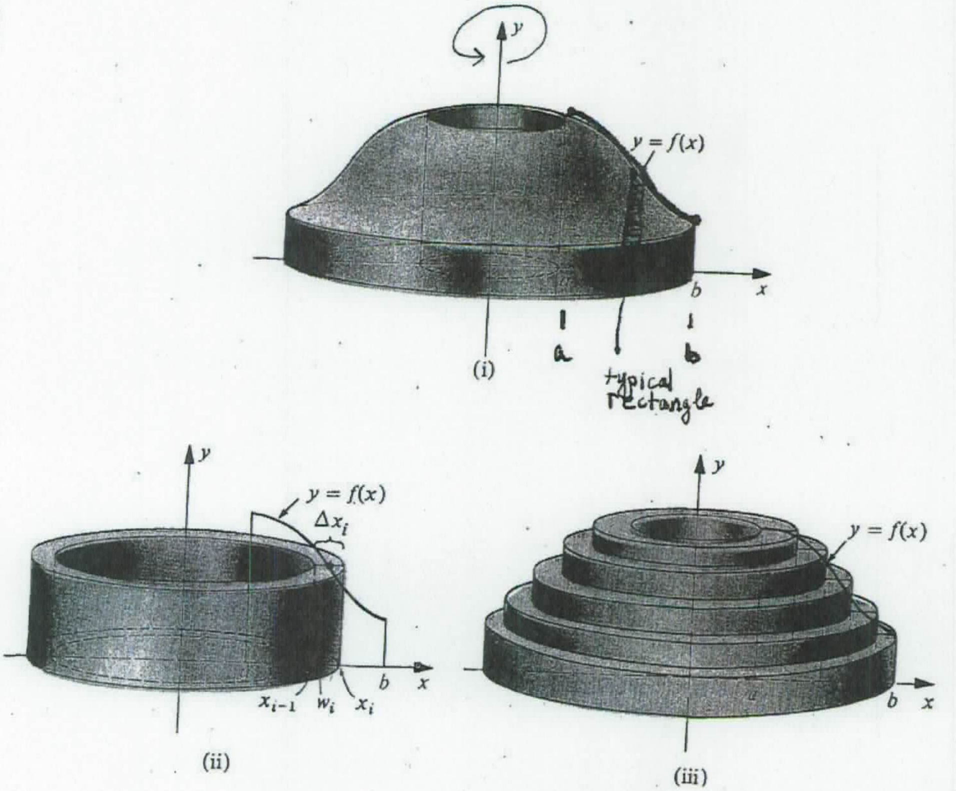
§6.3 Shell Method

Picture • our textbook pages 433-434

or

• Swoko p 290 (below)

Revolving about y-axis → partition x-axis



- Let's say partitioned z-axis (where $z = x$ or $z = y$)
- So $V = \int_{z=?}^{z=?} (\text{some function of } z)\, dz$

---

§6.2 **Disk/Washer Method : partition (∥-to) axis of revolution**

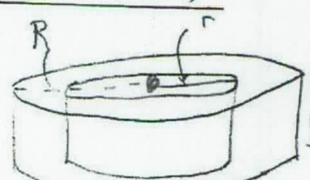
- typical element = disk or washer
- Disk (no hole)

$\} \Delta z = \text{height}$

Volume = (area of base) (height)

$= (\pi r^2)(\Delta z)$

- Washer (hole)



$\} \Delta z = \text{height}$

Volume = (Volume of big) − (Volume of little)

$= \pi R^2 \Delta z - \pi r^2 \Delta z$

$= \pi (R^2 - r^2) \Delta z$

$\cancel{=}\ \pi (R - r)^2 \Delta z$

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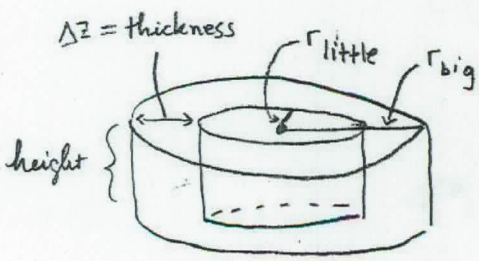
§6.3 **Shell Method : partition ⊥ to axis of revolution**

- typical element = shell.



Volume of typical shell

$= 2\pi$ (average radius) (height) (thickness)

$= 2\pi \left(\frac{r_{big} + r_{little}}{2}\right)(\text{height})\underbrace{(r_{big} - r_{little})}_{(\Delta z)}$

$= 2\pi$ ((avg) radius) (height) (Δz)