Start with an <u>INFINITE SERIES</u> $\sum_{n=1}^{\infty} a_n$. Think of it as

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots \qquad \hookleftarrow \qquad \text{the sum keeps on going and going - it is formal.}$$

Then form its corresponding <u>sequence</u> $\{s_N\}_{N=1}^{\infty}$ of <u>partial sums</u> where

$$s_N = \sum_{n=1}^{N} a_n = a_1 + a_2 + a_3 + \ldots + a_N \qquad \hookleftarrow \qquad \text{the sum stops at } N.$$

Then

$$\sum_{n=1}^{\infty} a_n \ \underline{\text{converges}} \text{ to } a \iff \lim_{N\to\infty} \sum_{n=1}^{N} a_n = a \iff \lim_{N\to\infty} s_N = a$$

and

$$\sum_{n=1}^{\infty} a_n \ \underline{\text{diverges}} \iff \lim_{N\to\infty} \sum_{n=1}^{N} a_n \text{ DNE} \iff \lim_{N\to\infty} s_N \text{ DNE.}$$

Rmk: $k=1$, doesn't matter where start

Algebraic Properties of Series

Let Σa_n and Σb_n be convergent series. Let c be a real number.

Then the following series converge: $\sum (ca_n)$ and $\sum (a_n + b_n)$ and $\sum (a_n - b_n)$.

Futhermore, they converge to what they should converge to, i.e.

$$\sum (ca_n) = c\left(\sum a_n\right) \quad \underline{\text{and}} \quad \sum (a_n + b_n) = \left(\Sigma a_n\right) + \left(\Sigma b_n\right) \quad \underline{\text{and}} \quad \sum (a_n - b_n) = \left(\Sigma a_n\right) - \left(\Sigma b_n\right)$$

kinda like a distributive property