I am standing 1 foot from the wall and am going to step towards the wall. With each step, I go half the distance remaining to the wall. How far do I go?

\[
\frac{1}{2} + \frac{1}{2} \left[ \text{of the distance remaining to the wall, which is } \frac{1}{2} \right] \\
+ \frac{1}{2} \left[ \text{of the distance remaining to the wall, which is } \left( \frac{1}{2} \right)^2 \right] \\
+ \frac{1}{2} \left[ \text{of the distance remaining to the wall, which is } \left( \frac{1}{2} \right)^3 \right] \\
+ \frac{1}{2} \left[ \text{of the distance remaining to the wall, which is } \left( \frac{1}{2} \right)^4 \right] + \ldots \quad \leftarrow \quad \text{the sum keeps on going}
\]

\[
= \left( \frac{1}{2} \right) + \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^3 + \left( \frac{1}{2} \right)^4 + \ldots \quad \leftarrow \quad \text{the sum keeps on going}
\]

\[
= \left( \frac{1}{2} \right) + \left( \frac{1}{4} \right) + \left( \frac{1}{8} \right) + \left( \frac{1}{16} \right) + \ldots \quad \leftarrow \quad \text{the sum keeps on going}
\]

\[
= \sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^n
\]

now let \( a_n = \left( \frac{1}{2} \right)^n \)

\[
= \sum_{n=1}^{\infty} a_n
\]

= ??? what ???

= ok, we see the answer

= but why, what do we REALLY mean by this INFINITE SUM ????

well \( \sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^n \) we can look at it this way \( \lim_{N \to \infty} \left[ \sum_{n=1}^{N} \left( \frac{1}{2} \right)^n \right] \)

takes some algebra \( \lim_{N \to \infty} \left[ 1 - \left( \frac{1}{2} \right)^N \right] \) from our knowledge of sequences \( 1 - 0 = 1 \)