Example 1 : against the wall

I am standing 1 foot from the wall and am going to step towards the wall.
With each step, I go half the distance remaining to the wall.
How far do I go?

$$\frac{1}{2} + \frac{1}{2}\left[\text{of the distance remaining to the wall, which is } \frac{1}{2}\right]$$

$$+ \frac{1}{2}\left[\text{of the distance remaining to the wall, which is } \left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\right]$$

$$+ \frac{1}{2}\left[\text{of the distance remaining to the wall, which is } \left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\right] + \ldots \quad \hookleftarrow \quad \text{the sum keeps on going}$$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \ldots \quad \hookleftarrow \quad \text{the sum keeps on going}$$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{4}\right) + \left(\frac{1}{8}\right) + \left(\frac{1}{16}\right) + \ldots \quad \hookleftarrow \quad \text{the sum keeps on going}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$$

now let $a_n = \left(\frac{1}{2}\right)^n$

$$= \sum_{n=1}^{\infty} a_n$$

$=$??? what ???

$=$ ok, we see the answer

$=$ but why , what do we REALLY mean by this <u>INFINITE SUM</u> ????

$$\overset{\text{well}}{=} \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \overset{\text{we can look at it this way}}{=} \lim_{N\to\infty}\left[\sum_{n=1}^{N}\left(\frac{1}{2}\right)^n\right]$$

$$\overset{\text{takes some algebra}}{=} \lim_{N\to\infty}\left[1-\left(\frac{1}{2}\right)^N\right] \overset{\text{from our knowledge of sequences}}{=} 1 - 0 = 1$$