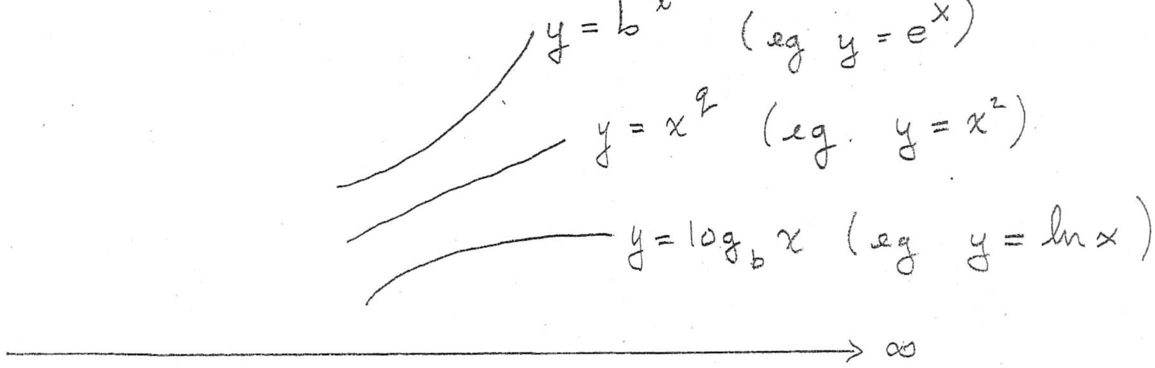
Series   Comparison Test

Helpful Intuition

For any power $0 < q < \infty$ and any base $b > 1$,
for n large enough

$$\log_b n \le n^q \le b^n .$$



In fact (L'Hopital's Rule)

$$\lim_{n\to\infty} \frac{\log_b n}{n^q} = 0 \quad \text{and} \quad \lim_{n\to\infty} \frac{n^q}{b^n} = 0.$$

So to figure out what happens to a series which involves a $\log_b n$ or b^n, remember

- $\log_b n$ grows "super slow" compared to n^q
- b^n grows "super fast" compared to n^q

Limit Comparison Test – beefed up.

40. (a) Suppose that $\Sigma\, a_n$ and $\Sigma\, b_n$ are series with positive terms and $\Sigma\, b_n$ is convergent. Prove that if

$$\lim_{n \to \infty} \frac{a_n}{b_n} = 0$$

then $\Sigma\, a_n$ is also convergent.

(b) Use part (a) to show that the series converges.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}\, e^n}$

§11.4 homework

41. (a) Suppose that $\Sigma\, a_n$ and $\Sigma\, b_n$ are series with positive terms and $\Sigma\, b_n$ is divergent. Prove that if

$$\lim_{n \to \infty} \frac{a_n}{b_n} = \infty$$

then $\Sigma\, a_n$ is also divergent.

(b) Use part (a) to show that the series diverges.

(i) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

↳ Prof G – explain to them!

↳ Students – can also do 40b & 41b with "Helpful Intuition"
– it's part of your homework ☺.