Series Flow Chart

$\sum_{n=17}^{\infty} a_n$

- ☐ absolutely convergent $\overset{\text{def}}{\Longleftrightarrow}$ $\left[\ \sum |a_n| \text{ conv.}\ \right]$ $\overset{\text{implies}}{\Longrightarrow}$ $\left[\ \sum a_n \text{ conv.}\ \right]$
- ☐ conditionally convergent $\overset{\text{def}}{\Longleftrightarrow}$ $\left[\ \sum |a_n| \text{ divg.} \quad \text{and} \quad \sum a_n \text{ conv.}\ \right]$
- ☐ divergent $\overset{\text{def}}{\Longleftrightarrow}$ $\left[\ \sum a_n \text{ divg.}\ \right]$ $\overset{\text{implies}}{\Longrightarrow}$ $\left[\ \sum |a_n| \text{ divg.}\ \right]$

Does $\sum |a_n|$ conv.?
Since $|a_n| \geq 0$, use a positive term test:
integral test, CT/LCT, ratio/root test.

$\overset{\text{if NO}}{\Rightarrow}$

$\sum |a_n|$ divg. so $\sum a_n$ is either cond. conv. or divg.

next $\Rightarrow$ try

$\lim_{n\to\infty} a_n = 0$?
or (∗)
equivalently
$\lim_{n\to\infty} |a_n| = 0$?

$\overset{\text{if NO}}{\Rightarrow}$

$\sum a_n$ divg. by n^{th} term test for divergence

if YES ⇓

$\sum a_n$ is abs. conv.

if YES ⇓

Is $\sum a_n$ an alternating series,
i.e., does $\sum a_n = \sum (-1)^n u_n$ where $u_n > 0$
i.e., does $a_n = (-1)^n u_n$ for some $u_n > 0$?

$\overset{\text{if NO}}{\Rightarrow}$

Is $\sum a_n$ is a telescoping series?

if YES ⇓ try AST

if YES ⇓

Does $\sum a_n \overset{\text{i.e.}}{=} \sum (-1)^n u_n$ satisfy the 1^{st} condition of the AST
i.e, are the u_n's decreasing,
i.e., is $u_{n+1} < u_n$?

find the partial sums $s_N = \sum_{n=1}^{N} a_n$
and then find $\lim_{N\to\infty} s_N$

if YES ⇓

Does $\sum a_n \overset{\text{i.e.}}{=} \sum (-1)^n u_n$ satisfy the 2^{nd} condition of the AST
i.e, does $\lim_{n\to\infty} u_n = 0$?
(you most likely already checked this at (∗) above since $|a_n| = u_n$)

if YES ⇓

$\sum a_n \overset{\text{i.e.}}{=} \sum (-1)^n u_n$ cond. conv.

$\overset{\text{if } \sum |a_n| \text{ divg.}}{\Longleftarrow}$

$\sum a_n \overset{\text{i.e.}}{=} \sum (-1)^n u_n$ converges by the AST

Useful: remember, in general,
$\lim_{n\to\infty} a_n = 0$
if and only if (∗∗)
$\lim_{n\to\infty} |a_n| = 0$