$\boxed{\text{Indeterminate Forms - L'Hôpital's Rule}}$

At $x = u$, $\boxed{}$ has the <u>indeterminate form</u>	$\boxed{}$ if $\lim\limits_{x \to u} f(x) =$	$\boxed{}$ and $\lim\limits_{x \to u} g(x) =$	$\boxed{}$
(1) $\frac{f(x)}{g(x)}$	$\frac{0}{0}$	0	0
(2) $\frac{f(x)}{g(x)}$	$\frac{\infty}{\infty}$	∞	∞
(3) $f(x) \cdot g(x)$	$0 \cdot \infty$	0	∞
(4) $f(x) - g(x)$	$\infty - \infty$	∞	∞
(5) $[f(x)]^{g(x)}$	0^0	0	0
(6) $[f(x)]^{g(x)}$	∞^0	∞	0
(7) $[f(x)]^{g(x)}$	1^∞	1	∞

HERE: u stands for any of the symbols a, a^-, a^+, $-\infty$, $+\infty$.

$\boxed{\text{L'Hôpital's Rule}}$

(1) and **(2)**

If:

- $\frac{f(x)}{g(x)}$ has the interdeterminate form $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$ at u

and

- $\lim\limits_{x \to u} \dfrac{f'(x)}{g'(x)}$ exists (i.e. this limit is a finite number or $-\infty$ or ∞)

then

$$\lim_{x \to u} \frac{f(x)}{g(x)} = \lim_{x \to u} \frac{f'(x)}{g'(x)} .$$

(3)

If $f(x) \cdot g(x)$ has the interdeterminate form $\boxed{0 \cdot \infty}$ at u, then rewrite:

$$f(x) \cdot g(x) = \frac{f(x)}{1/g(x)}$$, which has the interdeterminate form $\boxed{\frac{0}{0}}$ at u

or

$$f(x) \cdot g(x) = \frac{g(x)}{1/f(x)}$$, which has the interdeterminate form $\boxed{\frac{\infty}{\infty}}$ at u

and then apply L'Hôpital's Rule.

(4)

If $f(x) \; - \; g(x)$ has the interdeterminate form $\boxed{\infty \; - \; \infty}$ at u,

then use algebraic manipulation to convert $f(x) \; - \; g(x)$

into a form of the type $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$

and then apply L'Hôpital's Rule.

(5)

If $[f(x)]^{g(x)}$ has the interdeterminate form $\boxed{0^0}$ at u, then follow these steps:

Let

$$y = [f(x)]^{g(x)} \; .$$

So

$$\ln y = \ln \Big(\; [\; f(x) \;]^{g(x)} \; \Big) \; .$$

Next, simplify

$$\ln y = [g(x)] \; \cdot \; \ln [f(x)] \; .$$

Note that $\ln y = [g(x)] \; \cdot \; \ln [f(x)]$ has the interdeterminate form $\boxed{0 \; \cdot \; -\infty}$ at u.

Using an appropriate above method (i.e. **(3)**), evaluate

$$\lim_{x \to u} \; \ln y \; \equiv \; L \; .$$

Conclude

$$\lim_{x \to u} \; \ln [f(x)]^{g(x)} \; = \; L \qquad \Longrightarrow \qquad \lim_{x \to u} [f(x)]^{g(x)} \; = \; e^L \; .$$

(6) and **(7)**

If $[f(x)]^{g(x)}$ has the interdeterminate form $\boxed{\infty^0}$ or $\boxed{1^\infty}$, then proceed similarly as in **(5)** .

Note that $\ln y = [g(x)] \; \cdot \; \ln [f(x)]$ will have the interdeterminate form

(6) $\boxed{0 \; \cdot \; \infty}$ at u

(7) $\boxed{\infty \; \cdot \; 0}$ at u .