Math 142.501 Prof. Girardi

Fall 98 Final Exam 12/07/98

NAME: ______________________________

INSTRUCTIONS:

1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION,
      SHOW ALL YOUR WORK,
      INDICATE YOUR REASONING.
   b. when applicable put your answer on/in the line/box provided
   c. if no such line/box is provided, then box your answer
   d. if you use your calculator, give an explanation of what you did on it.
2. There are 18 pages to this exam: a total of 15 equally weighted problems.
   Check that your copy of the exam has all of the pages.
3. During this test, do not leave your seat.
   If you have a question, raise your hand.
   When you finish: turn your exam over, put your pencil down, raise your hand.
4. This closed book/notes exam covers (from *Calculus*, by Varberg and Purcell) :
   § 7.1 – 7.7 , 8.1 – 8.6 , 9.1 – 9.4 , 10.1 – 10.3 , 11.1 – 11.8 , 12.6 – 12.8 .

Problem Source:

1. Fall 1998 Exam 1 # 1c
2. Fall 1998 Exam 1 # 3, simplified
3. Fall 1998 Exam 2 # 1a
4. Fall 1998 Exam 2 # 1c
5. Fall 1998 Exam 2 # 2a, simplified
6. Fall 1998 Exam 2 # 4c
7. Fall 1998 Exam 3 # 3d
8. Fall 1998 Exam 3 # 3b
9. Fall 1998 Exam 4 # 1a
10. Example from class
11. Similar to homework problems § 11.7 # 1-8 and examples from class
12. Similar to examples from class
13. Fall 1998 Exam 2 # 5
14. Fall 1998 Exam 3 # 4
15. An example from class: § 8.4 # 85

bonus. the quote of our class!

BONUS:

Any fool can ______________________ , the point is to ______________________ .

______________________ Einstein

1. $\dfrac{d}{dx}\ e^{\tan x} =$ ____ .

2. $\dfrac{d}{dx} \; (\ln x)^{2x+3} =$ ____ .

HINT: of the form $\frac{d}{dx}\left([f(x)]^{g(x)}\right)$.

3. $\int \sin^3(2x)\, dx =$ ______________________________ + C .

4. $\displaystyle\int \frac{dx}{(16+x^2)^{3/2}} =$ ____ + C .

Your answer should NOT contain inverse trig functions . . . convert back .

5. $\displaystyle\int \ln x \, dx =$ ____ + C .

6. $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx \;=\;$ ________________________________ .

7. $\lim_{n\to\infty}(-1)^n \frac{100n}{n^{3/2}+4}$ = ____________________________________ .

8. $\lim_{n\to\infty}(-1)^n\frac{6n-5}{5n+1}$ = ____ .

9. $\sum_{n=7}^{\infty}(-1)^n \frac{n}{1+n^2}$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

10. Indicate **precisely** where the series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{n}$$

is absolutely convergent, conditionally convergent, and divergent (a diagram as we did in class suffices).

11. Using the Geometric Series, find a power series representation for

$$f(x) = \frac{7}{9 + 4x^2}$$

and specify precisely when (i.e., for what values of x's?) this representation is valid.

12. Grandpa and Grandma Puffo are out for their morning walk over a time period of $0 \leqslant \theta \leqslant 2\pi$. Grandpa is walking along a path given by $r = \sin\theta$ while Grandma is walking along a path given by $r = \sqrt{(3\cos^2\theta)}$.

12a. Fill in the below chart, similar to as we did in class.

θ	$r = \sin\theta$	$r = \sqrt{(3\cos^2\theta)}$
$0 \to \pi/2$	$\xrightarrow{1}$	$\xrightarrow{5}$
$\pi/2 \to \pi$	$\xrightarrow{2}$	$\xrightarrow{6}$
$\pi \to 3\pi/2$	$\xrightarrow{3}$	$\xrightarrow{7}$
$3\pi/2 \to 2\pi$	$\xrightarrow{4}$	$\xrightarrow{8}$

12b. Sketch below the curves of Grandpa and Grandma's walking paths, indicating parts 1–8, as we did in class

12c. GIVE <u>POINTS</u> IN POLAR FORM: (r, θ).
Grandpa and Grandma Puffo will pass eachother, and be able to give eachother

HIGH-FIVES, at time/times $\theta =$ ______________________________ .

At this time/these times, they will be at the point/points ________________ .

Grandpa and Grandma Puffo will pass through the same place, but not at the same time (so they will not be able to give eachother HIGH-FIVES), at point/points

____ .

13. Let $f\colon [0,14] \to \mathbb{R}$ and $g\colon [0,14] \to \mathbb{R}$ be two continuous functions, each differentiable on $(0,14)$. Also $g'(x) \neq 0$ for each $x \in (0,14)$. Cauchy Puffetto is moving along the curve $\mathcal{C}$ show below in such a way that his position at time t, where $0 \leqslant t \leqslant 14$, is (x,y) where $x = g(t)$ and $y = f(t)$. Estimate all points c, where $c \in (0,14)$, for which

$$\frac{f(14) - f(0)}{g(14) - g(0)} = \frac{f'(c)}{g'(c)} .$$

ESTIMATE(S) ON THE C('S): ______________________________ .
Clearly geometrically explain your reasoning! One point for the correct estimate(s) and 4 points for your explanation.

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14. Formally show (i.e., by using the definition of convergence: page 514) that

$$\lim_{n\to\infty} \frac{2n^3}{n^3+1} \;=\; 2\,.$$

Hints: I got you started, just fill in the missing parts. You might find helpful the observation that $\frac{2}{n^3+1} < \frac{2}{n^3}$.

Proof.

Recall that by definition of convergence of a sequence,

$$\lim_{n\to\infty} \frac{2n^3}{n^3+1} \;=\; 2$$

if and only if for each $\varepsilon > 0$ there exists a number $N_\varepsilon > 0$ such that if $n \geqslant N_\varepsilon$ then

$$\left| \frac{2n^3}{n^3+1} - 2 \right| \;<\; \varepsilon.$$

Let an arbitrary $\varepsilon > 0$ be given. Let $N_\varepsilon =$ ____________________. Then $n \geqslant N_\varepsilon$ implies that

$$\left| \frac{2n^3}{n^3+1} - 2 \right| \;=$$

Thus, by the definition of convergence of a sequence,

$$\lim_{n\to\infty} \frac{2n^3}{n^3+1} \;=\; 2\,. \qquad \blacksquare$$

15. Let $\mathbb{N}$ be the set of natural numbers, i.e.

$$\mathbb{N} \stackrel{\text{def}}{=} \{1, 2, 3, \dots\} .$$

Consider a function

$$f\colon [\,-\pi, \pi\,] \to \mathbb{R}$$

whose derivative

$$f'\colon [\,-\pi, \pi\,] \to \mathbb{R}$$

is continuous. For $n \in \mathbb{N}$, defined the n^{th} Fourier coefficient of f to be

$$a_n \stackrel{\text{def}}{=} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx \ .$$

Show that $\lim_{n\to\infty} a_n = 0$. Hint: integration by parts.

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More space for # 15: