MARK BOX

| Problem | Points | |
|---|---|---|
| 1 | 50 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| [redacted] | [redacted] | |
| Total | [redacted] 130 | |

MATH 142 sections 004 & 005
FALL 1993 Final Exam

KEY

$\boxed{1}$

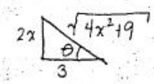
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | E | 6 | B | 11 | E | 16 | E | 21 | A | KEY |
| 2 | B | 7 | A | 12 | C | 17 | E | 22 | D | FINAL |
| 3 | B | 8 | B | 13 | A | 18 | D | 23 | C | MATH 142 |
| 4 | D | 9 | A | 14 | D | 19 | C | 24 | B | 93 FALL |
| 5 | D | 10 | D | 15 | A | 20 | A | 25 | C | |

2. Evaluate the following $\underline{2}$ integrals. ⊛ hint: +C ...

Pg 438 Ex 3 2a) $\displaystyle\int \frac{dx}{(4x^2+9)^2} = \underline{\frac{1}{108}\left[\tan^{-1}\left(\frac{2x}{3}\right) + \frac{2x}{\sqrt{4x^2+9}}\cdot\frac{3}{\sqrt{4x^2+9}}\right] + C}$

$2x = 3\tan\theta$

$\frac{2x}{3} = \tan\theta$




$$= \int \frac{\frac{3}{2}\sec^2\theta\, d\theta}{(9\tan^2\theta + 9)^2} = \frac{3}{2}\cdot\frac{1}{9^2}\int \frac{\sec^2\theta}{\sec^4\theta}\, d\theta$$

$$= \frac{1}{54}\int \cos^2\theta\, d\theta = \frac{1}{54}\cdot\frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{108}\left[\theta + \tfrac{1}{2}\sin 2\theta\right] + C = \frac{1}{108}\left[\theta + \tfrac{1}{2}\cdot 2\sin\theta\cos\theta\right] + C$$

From Exam #1 2b) $\displaystyle\int \ln(2x+7)\,dx = \underline{(x+\tfrac{7}{2})\ln(2x+7) - x + C}$

Parts $u = \ln(2x+7)$ $dv = dx$

$du = \frac{2\,dx}{2x+7}$ $v = x + \frac{7}{2}$

clever ∫

$$= (x+\tfrac{7}{2})\ln(2x+7) - \int \frac{2x+7}{2x+7}\,dx$$

$$= (x+\tfrac{7}{2})\ln(2x+7) - \int 1\,dx$$

3. Determine whether each of the following 2 series is absolutely convergent, conditionally convergent, or divergent. <u>CLEARLY</u> explain your reasoning and indicate the test(s) used. No credit will be given the work that does not make sense to us!!!!

Like #26 pg 561 3a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2+2}{n^3+3n}$

____ absolutely convergent
__X__ conditionally convergent
____ divergent

$\boxed{1^{st}}$ Consider $\sum \frac{n^2+2}{n^3+3n}$. L.C.T. with $b_n = \frac{n^2}{n^3} = \frac{1}{n}$ & $a_n = \frac{n^2+2}{n^3+3n}$

$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{n^2+2}{n^3+3n}\cdot\frac{n}{1} = \lim_{n\to\infty}\frac{n^3+2n}{n^3+3n} = 1$ $0 < 1 < \infty$

and $\sum b_n$ divg (harmonic series) so $\sum a_n$ divg.

$\boxed{2^{nd}}$ Consider $\sum(-1)^n\frac{n^2+2}{n^3+3n}$. Let $a_n = \frac{n^2+2}{n^3+3n}$ & $f(x) = \frac{x^2+2}{x^3+3x}$

A.S.T. 1. $a_n = f(n)$ $\forall n\in\mathbb{N}$

2. $a_n$ decreasing since $f'(x) = \frac{(2x)(x^3+3x) - (x^2+2)(3x^2+3)}{(x^3+3x)^2} = -\frac{(x^4+3x^2+6)}{(x^3+3x)^2} < 0$

3. $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{\frac{1}{n}+\frac{2}{n^3}}{1+\frac{3}{n^2}} = \frac{0}{1} = 0.$ $\Rightarrow$ AST $\Rightarrow \sum(-1)^n a_n$ conv

Like Ex 4 pg 554

3b) $\sum_{n=3}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

X absolutely convergent
___ conditionally convergent
___ divergent

1st Consider $\Sigma \frac{1}{n(\ln n)^2}$ & use integral test.

Let $f(x) = \frac{1}{x(\ln x)^2}$ • continuous for $x \geq 3$ • decreasing for $x \geq 3$

$\int_3^{\infty} \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty} \int_3^b \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty} \left[-\frac{1}{\ln x}\Big|_3^b\right]$

$= \lim_{b\to\infty} \frac{1}{\ln x}\Big|_b^3 = \lim_{b\to\infty} \frac{1}{\ln 3} - \frac{1}{\ln b} = \frac{1}{\ln 3} < \infty,$

So $\Sigma \frac{1}{n(\ln n)^2}$ converges.

4. Determine the interval of convergence for each of the following 2 power series. CLEARLY explain your reasoning. Do not forget to "check the endpoints."

4a) $\sum_{n=1}^{\infty} n^n (x-5)^n$ has interval of convergence $[5, 5]$ or $\{5\}$.

Root Test.

$\rho = \lim_{n\to\infty} |n^n (x-5)^n|^{1/n} = \lim_{n\to\infty} n \cdot |x-5| = \begin{cases} |x-5| \lim_{n\to\infty} n = \infty > 1 & \text{if } x \neq 5, \\ \lim_{n\to\infty} n \cdot 0 = \lim_{n\to\infty} 0 = 0 < 1 & \text{if } x = 5. \end{cases}$

Like problem on Exam 3

4b) $\sum_{n=1}^{\infty} \frac{(2x-6)^n}{10n+17}$ has interval of convergence $[\frac{5}{2}, \frac{7}{2})$.

Ratio Test

$\rho = \lim_{n\to\infty} \left| \frac{(2x-6)^{n+1}}{(2x-6)^n} \cdot \frac{10n+17}{10(n+1)+17} \right| = |2x-6| \lim_{n\to\infty} \frac{10n+17}{10n+27} = |2x-6| = 2|x-3| < 1$

$\Updownarrow$

$|x-3| < \frac{1}{2}$

Endpts.

$x = 3\frac{1}{2} = \frac{7}{2}$ : $\Sigma \frac{1}{10n+17}$ divg. LCT with $\frac{1}{n} = b_n$

$x = 2\frac{1}{2} = \frac{5}{2}$ $\Sigma \frac{(-1)^n}{10n+17}$ conv. A.S.T.

$\frac{1}{2}$ $\frac{1}{2}$ $2\frac{1}{2}$ $3$ $3\frac{1}{2}$

5. $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

5a) $1 - \cos t = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{t^{2n}}{(2n)!} = \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots$

5b) $\frac{1-\cos t}{t^2} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{t^{2n-2}}{(2n)!} = \frac{1}{2!} - \frac{x^2}{4!} - \frac{x^4}{6!} + \ldots$

5c) $\int_0^x \frac{1-\cos t}{t^2}\, dt = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{(2n)!\,(2n-1)}$ valid for all $x$.

Ex 2 pg. 586

5d) Using the above infinite series and the alternating series error estimate test, approximate $\int_0^1 \frac{1-\cos t}{t^2}\, dt$ within 5 decimal places of accuracy. You may leave your answer in the form of a sum of $n$ terms but explain why your choice of $n$ works!

$\int_0^1 \frac{1-\cos t}{t^2}\, dt \approx \frac{1}{2!} - \frac{1}{4!\,3} + \frac{1}{6!\,5} \approx .48639$.

⊛ Show your work below:

$\int_0^1 \frac{1-\cos t}{t^2}\, dt = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{(2n)!\,(2n-1)} = \underbrace{\frac{1}{2!}}_{n=1} - \underbrace{\frac{1}{4!\,3}}_{n=2} + \underbrace{\frac{1}{6!\,5}}_{n=3} - \underbrace{\frac{1}{8!\,7}}_{n=4}$

$\frac{1}{4!\,3} \approx .0138$ $\quad \frac{1}{6!\,5} \approx .0002778$ $\quad \frac{1}{8!\,7} \approx .000003$ 5 place