MARK BOX		
Problem	Points	
1	50	
2	20	
3	20	
4	20	
5	20	
Total	130	

MATH 142 sections 004 & 005
FALL 1993 Final Exam

NAME: ____________________

SSN: ____________________

Instructions:

(1) Expect for problem 1, to receive credit you must work in a logical fashion, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the line (or in the box) provided.
(2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.
(4) "Formula sheets" are not allowed. Calculators are allowed.
(5) This is a closed book/notes exam covering (from *Calculus* by Edwards & Penny) sections: 7.2–7.5, 8.2–8.3, 9.2–9.7, 11.1–11.4, 12.2–12.9, 10.1–10.2, 18.3, 10.4–10.6.

1. On a separate handout (which is yours to take home) are 25 multiple choice problems.
   (1) On the Scantron sheet provided, for each problem bubble one and only one selection.
   (2) Use a # 2 lead pencil.
   (3) Since our scantron machine is very sensitive, it is suggested that you first work through the handout, circling your choices directly on the handout, then go back and check your answers, then bubble in your scantron card.

   Prepare your scanton card as follows:
   (1) Print you name where indicated on the scantron card.
   (2) In the upper left-hand corner, bubble in you personal 4 digit number which starts with 00.
   (3) My personal 4 digit number is: | 0, 0, ______, ______ |.

MATH 142 sections 004 & 005
FALL 1993
Multile Choice portion of the Final Exam

Name ______________________

SSN ______________________

Multiple Choice. Select the correct answer and "bubble" your selection on the Scantron sheet provided. Use #2 pencil. Write your name and SSN on the Scantron sheet. Each question is worth 2 points, for a total of 50 points. (There are two extra BONUS questions.)

1. Which of the following is equal to ln 4 ?

(A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t\,dt$ (D) $\int_1^4 \ln x\,dx$ (E) $\int_1^4 \frac{1}{t}\,dt$

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2. $\int \tan(2x)\,dx =$

(A) $-2\ln|\cos(2x)| + C$ (B) $-\frac{1}{2}\ln|\cos(2x)| + C$ (C) $\frac{1}{2}\ln|\cos(2x)| + C$

(D) $2\ln|\cos(2x)| + C$ (E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

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3. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

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4. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

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5. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

---

6. If $f(x) = \ln(\sqrt{x})$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

---

7. $\int \sec^2 x\,dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

(D) $\frac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

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8. $\int_2^3 \frac{x}{x^2 + 1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

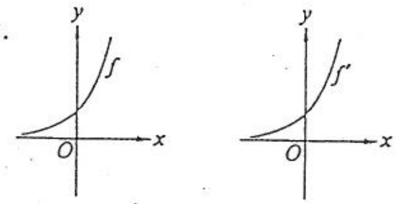
9. If $f(x) = e^x$, then $\ln[f'(2)] =$

(A) 2 (B) 0 (C) $\frac{1}{e^2}$ (D) $2e$ (E) e^2

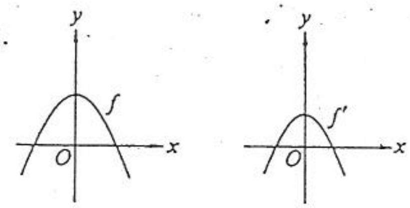
---

10. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?

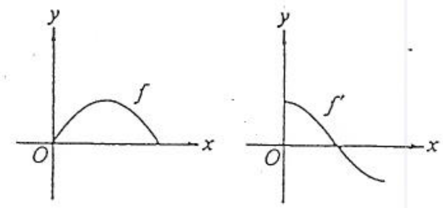
I.



II.



III.



(A) I only (B) II only (C) III only (D) I and III (E) II and III

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11. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

---

12. If $y = x^2e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x(x+2e^x)$ (C) $xe^x(x+2)$
(D) $2x+e^x$ (E) $2x+e$

---

13. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2}-1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2}-1)$ (E) $2(\sqrt{2}+1)$

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15. If $y = \text{Arctan}(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\text{Arcsec}(\cos x))^2 \sin x$ (C) $(\text{Arcsec}(\cos x))^2$

(D) $\frac{1}{(\text{Arccos}\, x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

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16. If $f(x) = (x^2+1)^x$, then $f'(x) =$

(A) $x(x^2+1)^{x-1}$

(B) $2x^2(x^2+1)^{x-1}$

(C) $x\ln(x^2+1)$

(D) $\ln(x^2+1) + \frac{2x^2}{x^2+1}$

(E) $(x^2+1)^x\left[\ln(x^2+1) + \frac{2x^2}{x^2+1}\right]$

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17. $\int_0^{\frac{\pi}{2}} x\cos x\,dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1-\frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2}-1$

---

18. $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$

(A) $-\frac{33}{20}$ (B) $-\frac{9}{20}$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln\left(\frac{8}{5}\right)$ (E) $\ln\left(\frac{2}{5}\right)$

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19. $\int_0^2 \sqrt{4-x^2}\,dx =$

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

20. $\int_2^{+\infty} \frac{dx}{x^2}$ is

(A) $\frac{1}{2}$ (B) $\ln 2$ (C) 1 (D) 2 (E) nonexistent

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21. If k is a positive integer, then $\lim\limits_{x \to +\infty} \frac{x^k}{e^x}$ is

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

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22. $\lim\limits_{n \to \infty} \frac{4n^2}{n^2 + 10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

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23. The Foci of the hyperbola with equation

$$4y^2 - x^2 - 24y + 14x = ~~18~~ \; 17$$

are
A. $(3 \pm \sqrt{5}, 7)$ B. $(3 \pm 2, 7)$ C. $(7, 3 \pm \sqrt{5})$
D. $(7, 3 \pm 1)$ E. $(7, \pm\sqrt{5})$.

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24. $\sin(2x) =$

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!} + \ldots$

(B) $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!} + \ldots$

(C) $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^n(2x)^{2n}}{(2n)!} + \ldots$

(D) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

(E) $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

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25. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges?

(A) $-1 \leqq x \leqq 1$ (B) $-1 < x \leqq 1$ (C) $-1 \leqq x < 1$

(D) $-1 < x < 1$ (E) All real x

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2. Evaluate the following 2 integrals. ⊛ hint: +C ...

2a) $\int \frac{dx}{(4x^2+9)^2}$ = ______________________________

2b) $\int \ln(2x+7)\,dx$ = ______________________________

⊛ show your work on the back of the previous page.

3. Determine whether each of the following 2 series is absolutely convergent, conditionally convergent, or divergent. CLEARLY explain your reasoning and indicate the test(s) used. No credit will be given the work that does not make sense to us!!!!

3a) $\sum_{n=1}^{\infty}(-1)^n \frac{n^2+2}{n^3+3n}$

____ absolutely convergent
____ conditionally convergent
____ divergent

3b) $\sum_{n=3}^{\infty} \frac{(-1)^n}{n\,(\ln n)^2}$

____ absolutely convergent
____ conditionally convergent
____ divergent

⊛ show your work on the back of the previous page.

4. Determine the interval of convergence for each of the following 2 power series. CLEARLY explain your reasoning. Do not forget to "check the endpoints."

4a) $\sum_{n=1}^{\infty} n^n (x-5)^n$ has interval of convergence ____________________.

4b) $\sum_{n=1}^{\infty} \frac{(2x-6)^n}{10n+17}$ has interval of convergence ____________________.

⊛ show your work on the back of the previous page.

5. Recall the power series expansion for $\cos x$, which is valid for all x:

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} .$$

Using this expansion and working through the steps below, find a power series expansion for the definite integral in part c). For what values of x is this series valid? Express your answers in closed form (i. e. using the $\sum$-sign).

5a) $1 - \cos t = \sum_{n=}$

5b) $\dfrac{1 - \cos t}{t^2} = \sum_{n=}$

5c) $\displaystyle\int_0^x \frac{1 - \cos t}{t^2}\, dt = \sum_{n=}$ valid for ______________.

5d) Using the above infinite series and the alternating series error estimate test, approximate $\displaystyle\int_0^1 \frac{1 - \cos t}{t^2}\, dt$ within 5 decimal places of accuracy. You may leave your answer in the form of a sum of n terms but explain why your choice of n works!

$\displaystyle\int_0^1 \frac{1 - \cos t}{t^2}\, dt \approx$ ______________________________.

⊛ Show your work below: