

MARK BOX	
Problem	Points
1	30
2	15
3	20
4	20
5	15
Total	100

MATH 142 sections 004 & 005
FALL 1993 EXAM # 3

NAME: _____
SSN: _____

Instructions:

- (1) To receive credit, you must work in a logical fashion, **SHOW ALL YOUR WORK**, **INDICATE YOUR REASONING**, and when applicable put your answer on the line (or in the box) provided.
- (2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.
- (4) "Formula sheets" are not allowed. Calculators are allowed.
- (5) This is a closed book/closed notes exam covering (from Calculus by Edwards & Penny) sections 12.7-12.9, 10.1-10.2.

1. Find the interval of convergence for each of the below power series. Do not forget to "check the endpoints." Has parts a), b) and c).

a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ has interval of convergence $(-\infty, \infty)$

$$\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = |x| \cdot \lim_{n \rightarrow \infty} \frac{1}{n+1} = |x| \cdot 0 < 1.$$

b) $\sum_{n=1}^{\infty} \frac{(2n)! x^n}{n!}$ has interval of convergence $[0, 0]$ or $\{0\}$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2n+2)!}{(2n)!} \cdot \frac{x^{n+1}}{x^n} \cdot \frac{n!}{(n+1)!} \right| = |x| \lim_{n \rightarrow \infty} \frac{(2n+1)(2n+2)}{(n+1)} =$$

$$= |x| \lim_{n \rightarrow \infty} \frac{4n^2 + 6n + 2}{n+1} = |x| \lim_{n \rightarrow \infty} \frac{4 + \frac{6}{n} + \frac{2}{n^2}}{1 + \frac{1}{n}} = |x| \cdot \infty$$

c) $\sum_{n=1}^{\infty} \frac{(2x-6)^n}{10n+17}$ has interval of convergence $\left[-\frac{5}{2}, \frac{7}{2} \right)$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2x-6)^{n+1}}{(2x-6)^n} \cdot \frac{10n+17}{10(n+1)+17} \right| = |2x-6| \lim_{n \rightarrow \infty} \frac{10n+17}{10n+27} = |2x-6|$$

1. Conv when $|2x-6| < 1 \iff |x-3| < \frac{1}{2}$

2. $\sum_{n=1}^{\infty} \frac{1}{10n+17}$ div (harmonic series) $\parallel \parallel \sum_{n=1}^{\infty} \frac{1}{10n+17} < \sum_{n=1}^{\infty} \frac{1}{10n} < \sum_{n=1}^{\infty} \frac{1}{n}$ Conv. Alt. Ser. Test

2. For each of the following functions, write a power series expansion (namely the Taylor series) about the point $a = 0$. Write your answer in closed form (i. e. using the \sum sign). Indicate the values of x for which the expansion is valid.

p 575 a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ valid for $|x| < \infty$

p 575 b) $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ valid for $|x| < \infty$

p 575 c) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ valid for $|x| < \infty$

Geometric Series d) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ valid for $|x| < 1$

3. Find a power series expansion (namely the Taylor series) of the below functions about the point a . Be clever and use your answers to question 2. Write your answer in closed form. Indicate the values of x for which the expansion is valid.

2 p 573 a) About $a = 0$: $e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ valid for $|x| < \infty$

1.14 # 19 p 573 c) About $a = 3$: $\frac{1}{21+x} = \sum_{n=0}^{\infty} \left(\frac{x-3}{18} \right)^n$ valid for $|x-3| < 18$

$$\frac{1}{21+x} = \frac{1}{18-(x-3)} = \frac{1}{18} \cdot \frac{1}{1-\frac{x-3}{18}}$$

5. On the same grid, sketch the curves $r = \sin \theta$ and $r^2 = 3 \cos^2 \theta$. The points (r, θ) , with θ in radians, of intersection of these two curves are

$$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3} \right), \left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3} \right), (0, 0)$$

44. The graph of the circle with polar equation $r = \sin \xi$ is shown as a dashed line in the figure at the right, while the graph of the equation

$$r^2 = 3 \cos^2 \xi$$

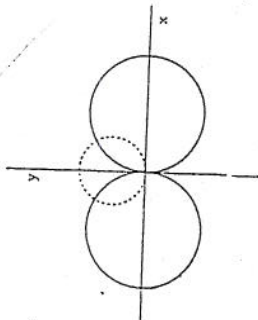
is shown as a solid line. The three points of intersection are

$$r = \frac{1}{2}\sqrt{3}, \quad \xi = \pi/3,$$

$$r = \frac{1}{2}\sqrt{3}, \quad \xi = 2\pi/3,$$

and

$$r = 0.$$



4. Working through the steps below, find a power series representation for the given definite integral. For what values of x is this series valid? Express your answers in closed form (i. e. using the \sum -sign).

a) $\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n$

b) $\frac{1}{1+t^5} = \sum_{n=0}^{\infty} (-t^5)^n = \sum_{n=0}^{\infty} (-1)^n t^{5n}$ valid if $|t^5| < 1 \iff |t| < 1$

c) $\int_0^x \frac{1}{1+t^5} dt = \sum_{n=0}^{\infty} (-1)^n \frac{t^{5n+1}}{5n+1}$ valid for $|x| < 1$

- d) Using the above infinite series and the alternating series error estimate test, approximate $\int_0^{0.5} \frac{1}{1+t^5} dt$ within 3 decimal places of accuracy. You may leave your answer in the form of a sum of n terms but explain why your choice of n works!

$$\int_0^{0.5} \frac{1}{1+t^5} dt \approx \frac{1}{2} - \frac{1}{6} \left(\frac{1}{2} \right)^6$$

- e) Show your work below:

$$\int_0^x \frac{dt}{1+t^5} = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{5n} dt = \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{5n} dt = \sum_{n=0}^{\infty} (-1)^n \frac{t^{5n+1}}{5n+1} \Big|_{t=0}^{t=x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n+1}}{5n+1}$$

$$\int_0^{0.5} \frac{dt}{1+t^5} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{5n+1} \left(\frac{1}{2} \right)^{5n+1}$$

an alternating series!

$$= \frac{1}{2} - \frac{1}{6} \left(\frac{1}{2} \right)^6 + \frac{1}{11} \left(\frac{1}{2} \right)^{11} - \dots$$

.0026
- .000044
A power series