MARK BOX		
Problem	Points	
1	10	
2	10	
3	40	
4	20	
5	20	
Total	100	

MATH 142 sections 004 & 005
FALL 1993 EXAM # 2

NAME: ______________________________

SSN: ______________________________

Instructions:

(1) To receive credit, you must work in a logical fashion, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the line (or in the box) provided.

(2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.

(4) "Formula sheets" are not allowed. Calculators are allowed.

(5) This is a closed book/closed notes exam covering (from *Calculus* by Edwards & Penny) sections 11.1–11.4, 12.2–12.6.

1. Let c be a fixed constant. Find the limit of the following sequence. Work carefully & indicate the reasoning behind your answer.

a) $\lim_{n\to\infty} \left[1 + \frac{c}{n}\right]^n = \boxed{}$

2. Evaluate the following integral. Work carefully & indicate the reasoning behind your answer.

$$\int_0^2 \frac{dx}{(x-1)^3} \quad = \quad \boxed{}$$

3. Determine whether each of the following 4 series is absolutely convergent, conditionally convergent, or divergent. <u>CLEARLY</u> explain your reasoning and indicate the test(s) used. No credit will be given the work that does not make sense to us!!!!

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{-2}{3}\right)^n$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n}{3^n-1}$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

c) $$\sum_{n=1}^{\infty} (-1)^n \, \frac{n^{\frac{1}{3}}}{n+1}$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

d) $$\sum_{n=2}^{\infty} \left[\ln \left(1 \; + \; \frac{1}{n} \right) \right]^n$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

4. Mr. Taylor

a) Find the 3rd degree Taylor polynomial $P_3(x)$ and the remainder term $R_3(x)$ about the point $a = 1$ for the function $f(x) = \ln x$.

Answer: $P_3(x) =$ $\boxed{}$

$R_3(x) =$ $\boxed{}$ where z is between

$\boxed{}$.

b) In part (a), to how many decimal places of accuracy does Taylor's formula guarantee that $P_3(x)$ approximate $f(x) = \ln x$ for x between .8 and 1.2? Show your work on the back of the previous page.

Answer: $\boxed{}$ decimal places of accuracy

5. A thinker . . . .

Fill in the blanks in the below statement of the Integral Test & Remainder Estimate: Let $\sum a_n$ be a positive-term series. Find a function $f(x)$ such that

(1) ______________________________

(2) ______________________________

(3) ______________________________ .

Then the series $\sum_{n=1}^{\infty} a_n$ and the improper integral $\int_1^{\infty} f(x)\,dx$ either:

(1) both ______________________________

(2) both ______________________________ .

If they both converge, then $\sum_{n=1}^{\infty} a_n = S_N \ + \ R_N$ where

$$S_N \equiv \sum_{n=1}^{N} a_n \qquad \text{and} \qquad R_N \equiv \sum_{n=N+1}^{\infty} a_n \ .$$

Furthermore,

$$\boxed{} \ \leq \ R_N \ \leq \ \boxed{} \ .$$

Now consider a sequence $\{b_n\}$ of positive numbers. We say that the *infinite product*

$$\prod_{n=1}^{\infty} (1 + b_n)$$

converges to P provided the associated *sequence* $\{P_N\}_{N=1}^{\infty}$ of *partial products* where

$$P_N \ \equiv \ \prod_{n=1}^{N} (1 + b_n)$$

converges to P. So "$\prod_{n=1}^{\infty}(1 + b_n)$ converges to P" is equivalent to

$$\lim_{N\to\infty} P_N \ = \ P$$

which is equivalent to

$$\lim_{N\to\infty} \ln(P_N) \ = \ \ln(P)$$

which is equivalent to

$$\lim_{N\to\infty} \sum_{n=1}^{N} \ln(1 + b_n) \ = \ \ln(P) \ .$$

Using (carefully) the Integral Test, determine whether or not the infinite series

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$$

converges. From the Integral Test Remainder Estimate, what can you conclude about the relation between $P_N \equiv \prod_{n=1}^{N} \left(1 + \frac{1}{n^2}\right)$ and $P \equiv \prod_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$? Show your work on the next page.