MATH 142 FALL 91 EXAM # 4 Outline to Answers

1a. $\sum_1^\infty \frac{x^n}{n!}$ converges for x in $(-\infty,\infty)$. Ratio Test $\rho = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n\to\infty} \frac{|x|}{n+1} = 0 < 1$.

1b. $\sum_1 \frac{n!\,x^n}{10^n}$ conv. only for $\boxed{x=0}$, i.e. $\boxed{x \in [0,0]}$. Ratio Test $\rho = \lim_{n\to\infty} \left| \frac{(n+1)!\,x^{n+1}}{10^{n+1}} \cdot \frac{10^n}{n!\,x^n} \right| = \lim_{n\to\infty} \frac{|x|}{10} \cdot (n+1) = \begin{cases} 0 < 1 & \text{if } x=0 \\ \infty > 1 & \text{if } x \neq 0. \end{cases}$

1c. $\sum \frac{(2x-6)^n}{5^n} \equiv \sum_1 \frac{2^n (x-3)^n}{5^n}$ conv. for $x \in \boxed{\left(\frac{1}{2}, \frac{11}{2}\right)}$ $\qquad \frac{1}{2} \quad 3 \quad 5\frac{1}{2}$

Root and Ratio Test both give $\rho = \lim_{n\to\infty} \left| \frac{2(x-3)}{5} \right| = \frac{2|x-3|}{5} < 1 \iff |x-3| < \frac{5}{2} = 2\frac{1}{2}$

Check endpts: $x = \frac{11}{2}$, have $\sum_1^\infty 1$ which divg. when $x = \frac{1}{2}$, have $\sum (-1)^n$ which divg.

2a. $e^t = \sum_{n=0}^\infty \frac{t^n}{n!} = 1 + t + \frac{t^2}{2!} + \dots$, 2b. $1 - e^t = 1 - \sum_{n=0}^\infty \frac{t^n}{n!} = \sum_{n=1}^\infty \frac{-t^n}{n!} = -t - \frac{t^2}{2!} - \dots$

2c. $\frac{1-e^t}{t} = \sum_{n=1}^\infty \frac{-t^n}{t(n!)} = \sum_{n=1}^\infty \frac{-t^{n-1}}{n!} = -1 - \frac{t^1}{2!} - \frac{t^2}{3!} - \dots$ $\qquad$ valid for all t so valid for all x

2d. $\int_0^x \frac{1-e^t}{t}\,dt = \int_0^x \sum_{n=1}^\infty \frac{-t^{n-1}}{n!}\,dt = \sum_{n=1}^\infty \int_0^x \frac{-t^{n-1}}{(n-1)!}\,dt = \sum_{n=1}^\infty \frac{-1}{(n-1)!} \int_0^x t^{n-1}\,dt = \sum_{n=1}^\infty \frac{-1}{(n-1)!} \frac{t^n}{n} \Big|_{t=0}^{t=x}$

$= \sum_{n=1}^\infty \frac{-1}{(n-1)!} \left[\frac{x^n}{n} - 0 \right] = \boxed{\sum_{n=1}^\infty \frac{-x^n}{(n-1)!\,n}}$

(3) Use alternating Series Estimate test

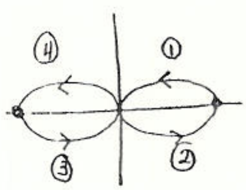
$\therefore$ not enough $\leftarrow$ 1 place $\leftarrow .008 \approx$ $\qquad \approx .0002$ 3 places enough.

3. $\sin x = \sum_{n=0}^\infty \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ for any x so let $x = 1^{\text{rad}}$ to see $\sin 1 = \sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)!} = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots$

so $\boxed{\sin 1^{\text{rad}} \approx 1 - \frac{1}{3!} + \frac{1}{5!}}$ within $\frac{1}{7!} \approx .0002$ so within 3 places so within 2 places.

4. $r^2 = 4\cos\theta$ $\quad \frac{1}{4}$ period $= \frac{1}{4} \cdot \frac{2\pi}{1} = \frac{\pi}{2}$

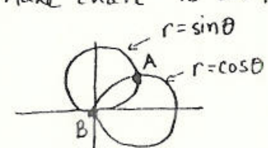
θ	$4\cos\theta$	$r = +\sqrt{}$	$r = -\sqrt{}$
$0 \to \frac{\pi}{2}$	$4 \to 0$	(1) $2 \to 0$	(3) $-2 \to 0$
$\frac{\pi}{2} \to \pi$	$0 \to -4$	no pts	no pts
$\pi \to \frac{3\pi}{2}$	$-4 \to 0$	no pts	no pts
$\frac{3\pi}{2} \to 2\pi$	$0 \to 4$	(2) $0 \to 2$	(4) $+0 \to -2$



5. $r = \sin\theta$ so $r^2 = r\sin\theta$ so $x^2 + y^2 = y$ so circle

$r = \cos\theta$ so $r^2 = r\cos\theta$ so $x^2 + y^2 = x$ so circle.

Make chart to see:



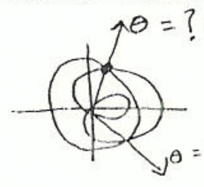
pts of $\cap$: $\boxed{B = (0,0)}$

$\sin\theta = \cos\theta \Rightarrow \theta = \frac{\pi}{4}$ $\quad r = \frac{\sqrt{2}}{2}$

same pt $\quad \theta = \frac{3\pi}{4}$ $\quad r = -\frac{\sqrt{2}}{2}$

So $A = \left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$

6.



pt of $\cap$: $2 = 1 + 2\cos\theta \iff \cos\theta = \frac{1}{2}$ so $\theta = \frac{\pi}{3}$ or $-\frac{\pi}{3}$

Symmetry

$A = \frac{1}{2} \int_\alpha^\beta r^2\,d\theta = \frac{1}{2} \int_{-\pi/3}^{\pi/3} \left[(1 + 2\cos\theta)^2 - (2)^2 \right] d\theta = 2 \cdot \frac{1}{2} \int_0^{\pi/3} \left[(1+2\cos\theta)^2 - (2)^2 \right] d\theta$

7. $x = 5\cos t \Rightarrow \frac{x}{5} = \cos t$
$y = 3\sin t \Rightarrow \frac{y}{3} = \sin t$
$0 \le t \le \pi$

$\cos^2 t + \sin^2 t = 1 \Rightarrow \boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$ $\qquad$ ellipse upper half

$-1 \le \cos t \le 1 \Rightarrow -1 \le \frac{x}{5} \le 1 \Rightarrow -5 \le x \le 5$

$0 \le t \le \pi \Rightarrow t$ is in 1st or 2nd Q $\Rightarrow 0 \le \sin t \le 1 \Rightarrow 0 \le \frac{y}{3} \le 1 \Rightarrow 0 \le y \le 3$