MARK BOX		
Problem	Points	You
1	20	
2	20	
3	40	
4	10	
5	10	
Total	100	

MATH 142.1 FALL 1991 EXAM 3

NAME: ______________________________

SSN: ______________________________

Instructions:

(1) To receive credit, you must work in a logical fashion, show all your work, and when applicable put your answer in the box (or on the line) provided.
(2) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.
(3) No "formula sheets" allowed. No calculators allowed.
(4) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.

1. For each of the following functions, write a power series expansion (namely the Taylor series) about the point $a = 0$. Write your answer in closed form. Indicate the values of x for which the expansion is valid.

a) $e^x = \sum_{n=}$ valid for ______________

b) $\cos x = \sum_{n=}$ valid for ______________

c) $\sin x = \sum_{n=}$ valid for ______________

d) $\frac{1}{1-x} = \sum_{n=}$ valid for ______________

2. Find a power series expansion (namely the Taylor series) of the below functions about the point a. Be clever and use your answers to question 1. Write your answer in closed form. Indicate the values of x for which the expanion is valid. In b) and c), show your work.

   a) About $a = 0$: $\quad e^{2x} = \sum_{n=}$ $\quad$ valid for ________________

   b) About $a = 0$: $\quad \dfrac{1}{4 - x^2} = \sum_{n=}$ $\quad$ valid for ________________

   c) About $a = 1$: $\quad \dfrac{1}{5 + x} = \sum_{n=}$ $\quad$ valid for ________________

3. Determine whether each of the following 4 series is absolutely convergent, conditionally convergent, or divergent. <u>CLEARLY</u> explain your reasoning and indicate the test(s) used. No credit will be given the work that does not make sense to us!!!!

a) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

b) $$\sum_{n=1}^{\infty} \frac{n^2+8}{10n^3+7}$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

c) $$\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

d) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent

4. Find the least integer N such that

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \approx \sum_{n=1}^{N} \frac{(-1)^n}{n}$$

within an error of $.0001 \equiv \frac{1}{10,000}$.

⊛ Indicate the error estimate test which you are using.

⊛ Show that you checked all the necessary hypothesis to be able to apply the test.

⊛ ANSWER: $N =$ _______________ .

5. Give an example of a pair of convergent series $\sum a_n$ and $\sum b_n$ such that $\sum a_n b_n$ diverges.

   ⊛ Hint: consider alternating series.

   ⊛ Answer: In my example, $a_n =$ ________________ and $b_n =$ ________________ .