Instructions:

(1) To receive credit, you must work in a logical fashion, show all your work, and when applicable put your answer in the box (or on the line) provided.

(2) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.

(3) No “formula sheets” allowed. No calculators allowed.

(4) The “Mark Box” indicates the problems along with their points. Check that your copy of the exam has all of the problems.

1. For each of the following functions, write a power series expansion (namely the Taylor series) about the point $a = 0$. Write your answer in closed form. Indicate the values of $x$ for which the expansion is valid.

a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ valid for $\infty$

b) $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ valid for $\infty$

c) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ valid for $\infty$

d) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ valid for $|x| < 1$
2. Find a power series expansion (namely the Taylor series) of the below functions about the point \( a \). Be clever and use your answers to question 1. Write your answer in closed form. Indicate the values of \( x \) for which the expansion is valid. In b) and c), show your work.

a) About \( a = 0 \) :
\[
e^{2x} = \sum_{n=0}^{\infty} \text{valid for } \\
\]

b) About \( a = 0 \) :
\[
\frac{1}{4-x^2} = \sum_{n=0}^{\infty} \text{valid for } \\
\]

c) About \( a = 1 \) :
\[
\frac{1}{5+x} = \sum_{n=0}^{\infty} \text{valid for } \\
\]
3. Determine whether each of the following 4 series is absolutely convergent, conditionally convergent, or divergent. Clearly explain your reasoning and indicate the test(s) used. No credit will be given the work that does not make sense to us!!!!

a) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \]

[ ] absolutely convergent
[ ] conditionally convergent
[ ] divergent

b) \[ \sum_{n=1}^{\infty} \frac{n^2 + 8}{10n^3 + 7} \]

[ ] absolutely convergent
[ ] conditionally convergent
[ ] divergent
c) \[ \sum_{n=2}^{\infty} \frac{\ln n}{n^3} \]

- absolutely convergent
- conditionally convergent
- divergent

d) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \]

- absolutely convergent
- conditionally convergent
- divergent
4. Find the least integer $N$ such that

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \approx \sum_{n=1}^{N} \frac{(-1)^n}{n} \]

within an error of $0.001 \equiv \frac{1}{10,000}$.

1. Indicate the error estimate test which you are using.
2. Show that you checked all the necessary hypothesis to be able to apply the test.

\[ \text{ANSWER: } N = \phantom{0000} \]
5. Give an example of a pair of convergent series $\sum a_n$ and $\sum b_n$ such that $\sum a_nb_n$ diverges.

* Hint: consider alternating series.

* Answer: In my example, $a_n =$ __________ and $b_n =$ __________.