MATH 142.1
FALL 1991
EXAM 2 - part 2
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Instructions:

(1) To receive credit, you must work in a logical fashion, show all your work, and when applicable put your answer in the box (or on the line) provided.
(2) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.
(3) No "formula sheets" allowed. No calculators allowed.
(4) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.

1. Find the limits of the following sequences. Indicate the reasoning behind your answer.

a) $\lim_{n\to\infty} (-1)^n =$

b) $\lim_{n\to\infty} \left(\frac{-1}{2}\right)^n =$

c) $\lim_{n\to\infty} (-2)^n =$

d) $\lim_{n\to\infty} n\,(\sin n\pi) =$

e) $\lim_{n\to\infty} \frac{\cos n}{3^n} =$

2. Find the limits of the following functions. Indicate the reasoning behind your answer.

   a) $\lim\limits_{x \to 0^+} \dfrac{\sin x}{x^2} \quad =$

   b) $\lim\limits_{x \to 0^+} x^x \quad =$

3. Find $\displaystyle\int_{-1}^{1} \frac{dx}{x^2} \quad =$

4. Evaluate the following integrals.

a) $\displaystyle\int \frac{dx}{x(x-1)^2} \quad =$

b) $\displaystyle\int \frac{3x+7}{x^2+4x+5}\, dx \quad =$

⊛ show your work on the back of the previous page.
⊛ hint: split up into two integrals

5. Recently, we have often computed that

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x \;+\; \frac{1}{2}\ln|\sec x \;+\; \tan x| \;+C$$

Using this fact, find $\int \sec^5 x\,dx \quad =$

6. a) Find the total area of the region bounded inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

- ⊛ hint 1: make a rough sketch and use symmetry
- ⊛ hint 2: total area means to view the area under the x-axis as positive so that your answer will not be 0.
- ⊛ hint 3: you may use the formula $\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$.
- ⊛ hint 4: this is easier than you think ....

Answer: The total area bounded by an ellipse is:

b) What happens to the total area when the number a approaches the number b? Why does this not surprise you?